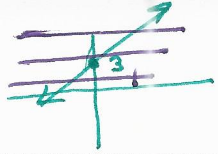


1-to-1 means for every y-value there is (at most) 1 x-value.

Mathematics 11

The Inverse of a Function

Date:

1) Find the inverse of the function $f(x) = 4x + 3$. Is the inverse a function?

Predict: f(x) is a linear function we know that f(x) passes HLT [f(x) is 1-to-1], that means the inverse will pass the VLT and be a fun-n.

Step 1: Replace f(x) with y: $y = 4x + 3$

Step 2: Interchange x and y: $x = 4y + 3$

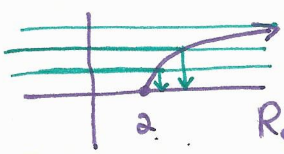
Step 3: Isolate the y: $4y = x - 3$, $y = \frac{x-3}{4} = \frac{1}{4}x - \frac{3}{4}$

Step 4: Replace y with $f^{-1}(x)$ (if inverse is a function)

$y = f^{-1}(x) = \frac{1}{4}x - \frac{3}{4}$

2) Find the inverse of $g(x) = \sqrt{x-2}$. Is the inverse a function?

g(x) is one-to-one. So the inverse of g(x) is a function.



$R_g = \{y \mid y \in \mathbb{R}, y \geq 0\}$

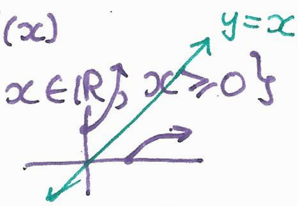
$y = \sqrt{x-2}$

$x = \sqrt{y-2}$

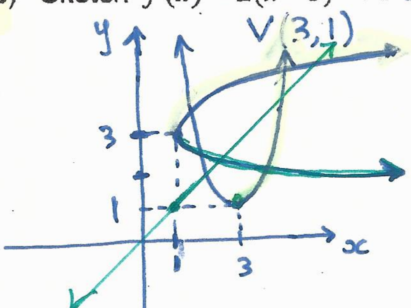
$x^2 = y - 2$

$y = x^2 + 2 = g^{-1}(x)$

$D_{g^{-1}} = R_g = \{x \mid x \in \mathbb{R}, x \geq 0\}$



3) Sketch $f(x) = 2(x-3)^2 + 1$ and its inverse. Restrict the domain of f so that the inverse is a function.



If $x \leq 3$ for $y = f(x)$

$f^{-1}(x) = -\sqrt{\frac{1}{2}(x-1)} + 3$, $y \leq 3$

$y = 2(x-3)^2 + 1$

$x = 2(y-3)^2 + 1$

$x - 1 = 2(y-3)^2$

$(y-3)^2 = \frac{x-1}{2}$

Take the sqrt of both sides.

$y - 3 = \pm\sqrt{\frac{1}{2}(x-1)}$

$y = \pm\sqrt{\frac{1}{2}(x-1)} + 3$

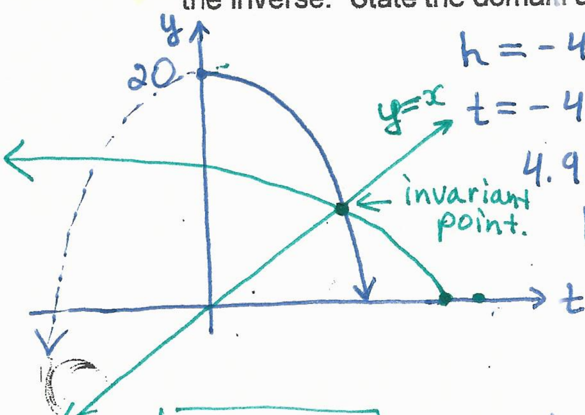
let $x \geq 3$ for $y = f(x)$ then $y \geq 3$ for inverse.

for $x \geq 3$ for $y = f(x)$ we have

$f^{-1}(x) = \sqrt{\frac{1}{2}(x-1)} + 3$

4) Word problem: The path of an object is modeled by $h(t) = -4.9t^2 + 20$. Sketch the function and find the inverse. State the domain and range of both h and its inverse.

$h(t) = -4.9(t-0)^2 + 20$; $V(0, 20)$



$h = -4.9t^2 + 20$

$t = -4.9h^2 + 20$

$4.9h^2 = -t + 20$

$h^2 = \frac{-t+20}{4.9} = \frac{-(t-20)}{4.9}$

$h = \pm\sqrt{-\frac{1}{4.9}(t-20)}$

In this situation: Given the time instant, the function h gives us the height of the object.

Given the height, the function h^{-1} gives us the time when the object is at that height.