Rational Exponents (Notes)

Review:

a) $\left(\frac{3}{4}\right)^{-2}$

b) $\frac{(-6)^0}{2^{-3}}$

c) $\frac{2^{-4}+2^{-6}}{2^{-3}}$

Rational Exponents (Basic Case - Roots)

$\sqrt[n]{\ }$ = n^{th} root   Example: $\sqrt[3]{64}$ cube root of 64, since $4^3 = 64$, therefore $\sqrt[3]{64} = 4$

Terminology $\sqrt[n]{x}$ where n = Index x = Radicand $\sqrt{\ }$ = Radical sign

$\sqrt[n]{x}$ = n^{th} root of $x = x^{\frac{1}{n}}$. If n is even, we require $x \geq 0$ i.e. $\sqrt[2]{-4}$ does not exist in real numbers

If n is odd, $x \in R$, for example $\sqrt[3]{-8} = -2$

Example 1: Evaluate

a) $64^{\frac{1}{2}}$

b) $(-27)^{\frac{1}{3}}$

c) $(-8)^{\frac{-1}{3}}$

Rational Exponents (General Case)

Recall the following laws of exponents $(a^m)^n = a^{m\times n}$ & $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$. Those can be applied to show that

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m \quad (m, n \in N)$$

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{(\sqrt[n]{a})^m} \quad (m, n \in N, a > 0)$$

Example: $4^{\frac{3}{2}} =$ and $(4^3)^{\frac{1}{2}} =$

Example 2: Evaluate

a) $(-8)^{\frac{4}{3}}$

b) $9^{-2.5}$

c) $\left(\frac{25}{4}\right)^{\frac{-3}{2}}$