Rational Exponents (Notes)

Review:

a) $\left(\frac{3}{4}\right)^{-2}$ b) $\frac{(-6)^0}{2^{-3}}$ c) $\frac{2^{-4}+2^{-6}}{2^{-3}}$

Rational Exponents (Basic Case - Roots)

$\sqrt[n]{\ }$ = nth root Example: $\sqrt[3]{64}$ cube root of 64, since $4^3 = 64$, therefore $\sqrt[3]{64} = 4$

Terminology $\sqrt[n]{x}$ where n = Index x = Radicand $\sqrt{\ }$ = Radical sign

$\sqrt[n]{x}$ = nth root of $x = x^{\frac{1}{n}}$. If n is even, we require $x \geq 0$ i.e. $\sqrt[2]{-4}$ does not exist in real numbers

If n is odd, $x \in R$, for example $\sqrt[3]{-8} = -2$

Example 1: Evaluate

a) $64^{\frac{1}{2}} = \sqrt{64} = 8$

b) $(-27)^{\frac{1}{3}} = \sqrt[3]{-27} = -3$

c) $(-8)^{\frac{-1}{3}} = \frac{1}{(-8)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{-8}} = \frac{1}{-2} = -\frac{1}{2}$

Rational Exponents (General Case)

Recall the following laws of exponents $(a^m)^n = a^{m\times n}$ & $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$. Those can be applied to show that

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ $(m, n \in N)$ positive integers $\frac{3}{2} = \frac{1}{2}\cdot\frac{3}{1}$

$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{(\sqrt[n]{a})^m}$ $(m, n \in N, a > 0)$

Example: $4^{\frac{3}{2}} = (4^{\frac{1}{2}})^3 = (\sqrt[2]{4})^3 = 2^3 = 8$ ✓ or $(4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} = \sqrt{64} = 8$ ✓

Example 2: Evaluate

a) $(-8)^{\frac{4}{3}} = \left[(-8)^{\frac{1}{3}}\right]^4$

$= (\sqrt[3]{-8})^4$

$= (-2)^4 = 16$

b) $9^{-2.5}$

$= 9^{-\frac{5}{2}}$

$= \frac{1}{9^{\frac{5}{2}}} = \frac{1}{(9^{\frac{1}{2}})^5} = \frac{1}{(\sqrt{9})^5}$

$= \frac{1}{3^5} = \frac{1}{243}$

c) $\left(\frac{25}{4}\right)^{\frac{-3}{2}}$

$= \left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{25}}\right)^3$

$= \left(\frac{2}{5}\right)^3 = \frac{8}{125}$