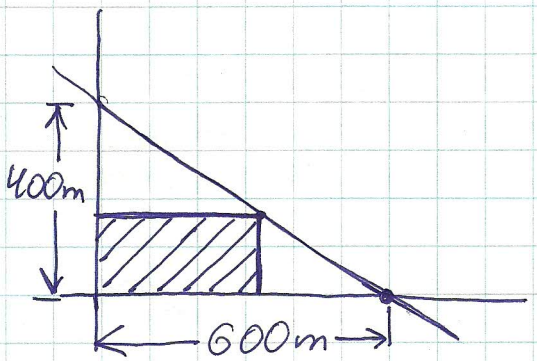
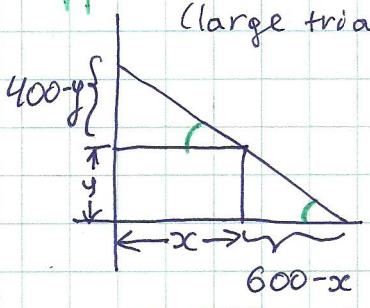
Quadratic Word Problems

(#17) A straigh section of railroad track crosses two highways 400m and 600m from their intersection. Find the dimensions of the largest rectangular lot that can be laid out in the triangle formed by the railroad and highways



Approach 1: Similar triangles

(large triangle and upper smaller)



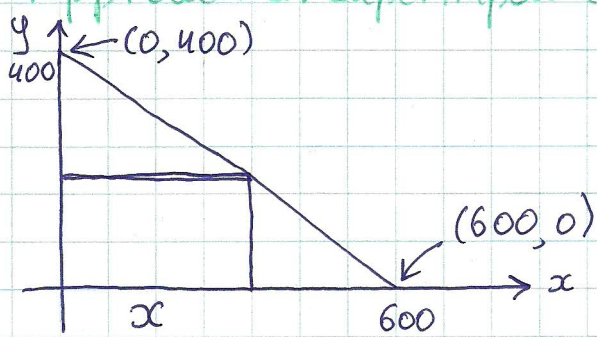
$$\frac{400}{600} = \frac{400-y}{x}$$

$$\frac{2}{3} = \frac{400-y}{x}$$

$$\frac{2}{3}x = 400 - y$$

$$\boxed{y = -\frac{2}{3}x + 400}$$

Approach 2: Superimpose Cartesian Plane



Find the equation of the railroad line: $y = mx + b$

$$m = \frac{400-0}{0-600} = \frac{400}{-600} = -\frac{2}{3}$$

$y = -\frac{2}{3}x + b$, Use $(0, 400)$: $400 = -\frac{2}{3}(0) + b$

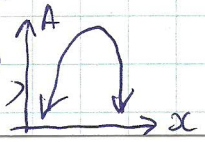
$b = 400$

$$\boxed{y = -\frac{2}{3}x + 400}$$

Either way, $A = x(y) = x\left(-\frac{2}{3}x + 400\right)$

$$A = -\frac{2}{3}x^2 + 400x$$

$$A = -\frac{2}{3}(x^2 - 600x) = -\frac{2}{3}(x^2 - 600x + 90000 - 90000) = -\frac{2}{3}(x-300)^2 + 60000$$

$a = -\frac{2}{3} < 0$ 

$\therefore$ The maximum value of A is 60000 when $x = 300$m and $y = 200$m.