Geometric Sequences.

Ex 1: (a) $2, 6, 18, 54, 162, \ldots$

multiplication pattern

$2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} 162, \ldots$

$$3 = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots = \frac{t_n}{t_{n-1}} = r \leftarrow \text{common ratio.}$$

$a = t_1 = 2$

$t_2 = 2 \cdot 3$

$t_3 = 2 \cdot 3^2$

$t_4 = 2 \cdot 3^3$

$t_5 = 2 \cdot 3^4$

$\Rightarrow t_n = 2 \cdot 3^{n-1}$ ✓

(b) $20, -10, 5, -\frac{5}{2}, \frac{5}{4}, \ldots$

$\frac{5}{4} \times \frac{-2}{5} = -\frac{2}{4} = -\frac{1}{2}$

$a = t_1 = 20$

$$r = \frac{t_n}{t_{n-1}} = \frac{-10}{20} = \frac{5}{-10} = \frac{-\frac{5}{2}}{5} = \frac{\frac{5}{4}}{-\frac{5}{2}} = -\frac{1}{2}$$

$t_n = 20\left(-\frac{1}{2}\right)^{n-1}$

$t_2 = 20\left(-\frac{1}{2}\right), \; t_3 = 20\left(-\frac{1}{2}\right)^2, \ldots$

(c) $a, ar, ar^2, ar^3, \ldots, \underbrace{ar^{n-1}}_{t_n}, \ldots$

$t_1 = a,$

$t_n = ar^{n-1}$

How many terms are there?

Ex 2: $5, -10, 20, -40, \ldots, 1280$

Geometric sequence:

$a = t_1 = 5$

$r = \frac{t_n}{t_{n-1}} = \frac{-40}{20} = -2$; $t_n = ar^{n-1}$

$t_n = 5(-2)^{n-1}$

Set $t_n = 1280$, $t_n = 1280 = 5(-2)^{n-1}$

$5(-2)^{n-1} = 1280$

$(-2)^{n-1} = \frac{1280}{5} \rightarrow (-2)^{n-1} = 256$

We want BATS, if possible.

$(-2)^{n-1} = (-2)^8$

BATS

$n - 1 = 8$

$n = 8 + 1$, $n = 9$

$t_n = ar^{n-1} = f(n)$ ← The exponential Function.

n	t_n	1st diff	Ratios
1	5		
		-15	
2	-10		-2
		30	
3	20		-2
		-60	
4	-40		-2
		120	
5	80		⋮
		-240	
6	-160		

Ex 3: Find t_{10} in the geometric sequence where $t_3 = 45$ and $t_7 = 3645$.

$t_n = ar^{n-1}$ $\quad a \neq 0$

$t_3 = 45 = ar^2$ ①

$t_7 = 3645 = ar^6$ ②

② ÷ ①: $\frac{3645}{45} = \frac{ar^6}{ar^2} \rightarrow 81 = r^4$

$\rightarrow r^4 - 81 = 0$

$(r^2 - 9)(r^2 + 9) = 0$

$r^2 - 9 = 0$ or $r^2 + 9 = 0$ (not possible in real numbers)

$(r-3)(r+3) = 0$

$r = 3$ or $r = -3$

Case 1: $t_n = 5(3)^{n-1}$

$r = 3$

$ar^2 = 45$

$a(3)^2 = 45$

$a = \frac{45}{9} = 5$

$t_{10} = ar^9$

$= 5(3)^9$

$= 98415$

Case 2: $r = -3$

$r = -3$ $\quad a(-3)^2 = 45$

$a = 5$

$t_n = 5(-3)^{n-1}$

$t_{10} = 5(-3)^9$

$t_{10} = -98415$

Recursion Formula:

$t_1 = a$

$t_n = t_{n-1} \cdot r$