

Equivalent fractions (rational expressions) are fractions in different form that have the same value when reduced. To reduce fractions we first find factors that are common to the numerator and denominator. Then we divide the numerator and the denominator by the same (common) quantity.

Example 1: Simplifying Fractions (Writing a fraction in simplest terms)

Find a reduced fraction equivalent to each of the following ones:

(a) $\frac{2x-10}{2x+8}$

(b) $\frac{3x-6}{3x+3}$

(c) $\frac{x^2-6x}{x^2+3x}$

(d) $\frac{-x+10}{-x-5}$

We can also write a fraction equivalent to a given one by writing it in a more involved form. This is done by multiplying the numerator and the denominator by the same non-zero quantity. The value of fraction is thus unchanged as we effectively multiply the fraction by one.

Indeed, $\frac{a}{a} = 1$ as long as $a \neq 0$.

Example 2: Multiplying a fraction by 1.

Given $\frac{2x+3}{5x}$

(a) Write an equivalent fraction by multiplying by $\frac{2}{2}$

(b) Write an equivalent fraction by multiplying by $\frac{x}{x}$

Example 3: Multiplying by one when there is more than one fraction.

Find the LCD for the given rational expressions, and convert the rational expressions into equivalent rational expressions with the LCD as the denominator

(a) $\frac{1}{3x^2}, \frac{3}{2x^5}$

(b) $\frac{b-a}{ab}, \frac{a-b}{b^2}$

(c) $\frac{x-1}{3x-12}, \frac{x-3}{2x-8}$

(d) $\frac{3x-8y}{x^2-2xy}, \frac{3xy-x}{xy-2y^2}$

Practice:

1. Fill in the blanks:

$$(a) \frac{a}{x-2} = \frac{\quad}{x^2-2x}$$

$$(b) \frac{a}{x-2} = \frac{\quad}{x^2-4}$$

$$(c) \frac{a}{x-2} = \frac{\quad}{6-3x}$$

$$(d) \frac{a}{x-2} = \frac{\quad}{(x-2)^2}$$

$$(e) \frac{a}{x-2} = \frac{\quad}{4-x^2}$$

$$(f) \frac{a}{x-2} = \frac{\quad}{(x-2)^3}$$

2. Fill in the blanks:

$$(a) \frac{x}{x-1} = \frac{\quad}{x^2-1}$$

$$(b) \frac{3}{a+2} = \frac{\quad}{a^2+4a+4}$$

$$(c) \frac{m-2}{m+2} = \frac{\quad}{4-m^2}$$

$$(d) \frac{y-1}{y+3} = \frac{\quad}{2y^2+5y-3}$$