Completing the Square.

Ex1: (Solving an eq-n)

$$2x^2+5x+1=0$$

$$2\left(x^2+\frac{5}{2}x\right)+1=0$$

$$2\left(x^2+\frac{5}{2}x+\frac{25}{16}-\frac{25}{16}\right)+1=0$$

$$2\left(x^2+\frac{5}{2}x+\frac{25}{16}\right)-\frac{25}{8}+\frac{8}{8}=0$$

$$2\left(x+\frac{5}{4}\right)^2-\frac{17}{8}=0$$

$$2\left(x+\frac{5}{4}\right)^2=\frac{17}{8}$$

$$\left(x+\frac{5}{4}\right)^2=\frac{17}{16}$$

$$x+\frac{5}{4}=\pm\sqrt{\frac{17}{16}}$$

$$x+\frac{5}{4}=\pm\frac{\sqrt{17}}{4}$$

$$x=-\frac{5}{4}\pm\frac{\sqrt{17}}{4}$$

$$x_{1,2}=\frac{-5\pm\sqrt{17}}{4}$$

$$\boxed{\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}}$$

$$\sqrt{ab}=\sqrt{a}\sqrt{b}$$

$$ax^2+bx+c=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\ ; \quad D=b^2-4ac$$

$\to x_1=\dots \qquad x_2=\dots$

$\boxed{\text{Ex 1:}}$

$2x^2-x+3=0$ How many solutions and of which type (real/non-real)?

The discriminant $D=(-1)^2-4(2)(3)=-23<0$

$\therefore$ no real solutions.

Two complex solutions.

$\downarrow$

$a+bi$ real # + pure imaginary #

$\downarrow a \qquad \downarrow bi$

ex: $3-2i$, $-5i$, $17+i$.

$\sqrt{-9}=\sqrt{-1\cdot 9}=\sqrt{-1}\sqrt{9}=3\sqrt{-1}=3i$

$\boxed{i=\sqrt{-1}}$ imaginary unity.

$\boxed{\text{Ex 2:}}$

$25x^2+60x+36=0$

$D=60^2-4(25)(36)=3600-3600=0$

$\therefore$ one distinct real solution.

$25x^2+60x+36$
$=(5x+6)^2=0$

$5x+6=0$

$x=-\frac{6}{5}$

or

$x=-\frac{6}{5}$

$\boxed{\text{Ex 3:}}$ $2x^2-11x+3=0$

$D=(-11)^2-4(2)(3)=121-24=\underline{\underline{97>0}}$

Two distinct real solutions.