p. 164 #4

The sum of a number and three times another number is 18. Determine the numbers if their product is a maximum.

① let P represent the product (in general, not maximum!) ← quantity to be maximized

let x represent the first number then

$x + 3y = 18$

② $P = x \cdot y$ ← Equation for quantity to be maximized.

③ $P = y(18 - 3y)$

$P = 18y - 3y^2$ Rewriting in standard form first.

④

$P = -3y^2 + 18y$

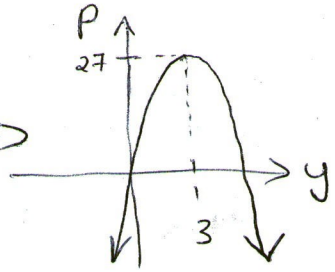
$P = -3(y^2 - 6y)$

$P = -3(y^2 - 6y + 9 - 9)$

$P = -3(y^2 - 6y + 9) + 27$

$P = -3(y - 3)^2 + 27$

completing the square.



⑤ The max value of P is 27 when $y = 3$ } max/min statement

⑥ Since the question is asking for both numbers

$y = 3$ and $x = 18 - 3y$

$x = 18 - 3(3)$

$x = 9$

Answer: 3 and 9.