

General Quadratic Equation $ax^2 + bx + c = 0$

standard form

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{4ac}{4a}$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

divide by $a \neq 0$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \checkmark$$

The Quadratic Formula.

Example: $2x^2 - 5x + 1 = 0$

$$a = 2 \quad b = -5 \quad c = 1$$

Substitution is a step! $x_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$

$$x_{1,2} = \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4}$$

$$x_1 = \frac{5 + \sqrt{17}}{4}, \quad x_2 = \frac{5 - \sqrt{17}}{4}$$

$$x_1 = \frac{5}{4} + \frac{\sqrt{17}}{4}, \quad x_2 = \frac{5}{4} - \frac{\sqrt{17}}{4}$$

conjugate pairs!

$$\begin{aligned} \textcircled{1} \quad \frac{b}{a} \cdot \frac{1}{2} &= \frac{b}{2a} \\ \textcircled{2} \quad \left(\frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} \end{aligned}$$

$$\frac{c}{1} = \frac{4ac}{4a}$$

$$\sqrt{4a^2} = 2a$$