

What can we say about a positive integer that has an even $t(n)$?

Solution:

$$n = p_1^{e_1} \cdot p_2^{e_2} \cdot \dots \cdot p_k^{e_k}$$

$$t(n) = (e_1 + 1)(e_2 + 1) \cdot \dots \cdot (e_k + 1)$$

If $t(n)$ is even, then at least one of $(e_1 + 1), (e_2 + 1), \dots, (e_k + 1)$ is even.

Say $e_1 + 1$ is even, $e_1 + 1 = 2m, m \in \mathbb{Z}$

$$e_1 = 2m - 1, \boxed{e_1 \text{ is odd}}$$

\therefore There is at least one odd exponent in PND of n .

Which means n cannot be a perfect square.

(n is a non-square)