Positive integers.

Prime and Composite Natural Numbers.

A ~~divisor~~ factor of a number is a natural number that goes evenly (without remainder) into the given number.

Example: number 20 has six divisors/factors, which are: 1, 2, 4, 5, 10, 20. $\boxed{t(20)=6}$

A natural number, other than 1, is called a prime number if it has only two divisors: 1 and the number itself. ✓
There exists the smallest prime number, which is: 2. This is the only prime number that is even.

Any even number greater than 2 is, therefore a composite number. There is no largest prime number as there are infinitely many prime numbers. A natural number that has more than two divisors is called a composite number.
proof?

Any composite number N is divisible by some number that is less than or equal to the square root of N.

Why? Divisors come in pairs. If N is a perfect square, then $\sqrt{N}$ is a divisor and other divisors are less or greater so that each divisor less than $\sqrt{N}$ is paired with a divisor greater than $\sqrt{N}$. Therefore, it enough to check numbers (prime) less then $\sqrt{N}$.

If N is a non-square, then $\sqrt{N}$ is not a divisor of N. The divisors of N are either less than $\sqrt{N}$ or greater than $\sqrt{N}$ and each divisor less than $\sqrt{N}$ is paired up with a divisor greater than $\sqrt{N}$.

That gives us a way of checking if a number is a prime.

Example: Check if 137 is a prime.

$11 < \sqrt{137} < 12$

~~2, 3, 5, 7, 11~~.

$\sqrt{137} \doteq ?$
$\doteq 11.?$

$121 < 137 < 144$

$\therefore$ 137 is a prime!

Problem:
Prime numbers have only two divisors: number 1 and the number itself.
What numbers have exactly 3 different divisors? ____________ ____________ __ ____________ .

[Hint: To every divisor m of the dividend M there corresponds another divisor $\frac{M}{m}$, in other words divisors come in pairs].
What numbers have exactly 4 different divisors?

Type 1:

Type 2:

The Fundamental Theorem of Arithmetic:
Any natural number N greater than 1 can be expressed as a product of primes.
This representation is unique, except for the order of factors. Then we have a record of the type
$N = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$

The number of prime divisors of N according to this record is ____. For example, $2^3 3^2 5^9$ has ___ prime divisors.

Example: Factorization of a Natural Number into Prime Factors.
Determine the prime number decomposition of 72.

(a) Factor Tree *(b) Column Method* *(c)Step Method*