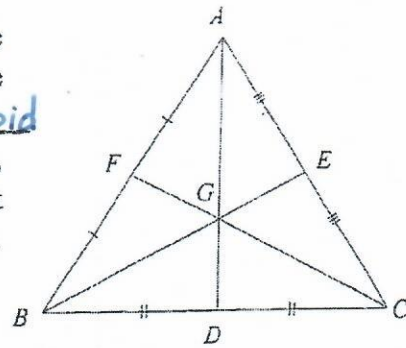


Geometric Facts

1. The centroid (The Centre of Mass)

The three medians (line from vertex to mid-point of the opposite side) of any triangle are concurrent (meet at the same point). The intersection point is called the centroid of the triangle, usually denoted by G . For each median, the length from the vertex to the centroid is twice that from the centroid to the mid-point of the opposite side, i.e.

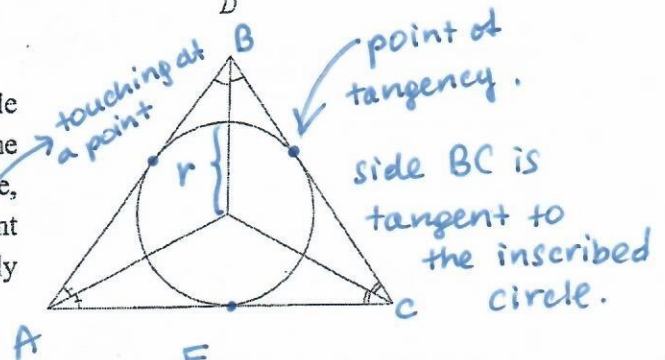
$$\underline{AG = 2GD}, \underline{BG = 2GE} \text{ and } \underline{CG = 2FG}$$



2. The in-centre

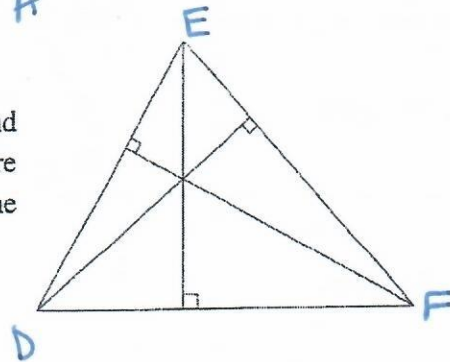
The three angle bisectors (line which divides the angle into two equal halves) of any triangle are concurrent. The intersection point is called the incenter of the triangle, and it is the centre of the inscribed circle (circle tangent to the three sides of the triangle) of the triangle, usually denoted by I .

radius = r



3. The orthocentre

The three altitudes (from vertex to the opposite side and perpendicular to the opposite side) of any triangle are concurrent. The intersection point is called the orthocentre of the triangle, usually denoted by H .

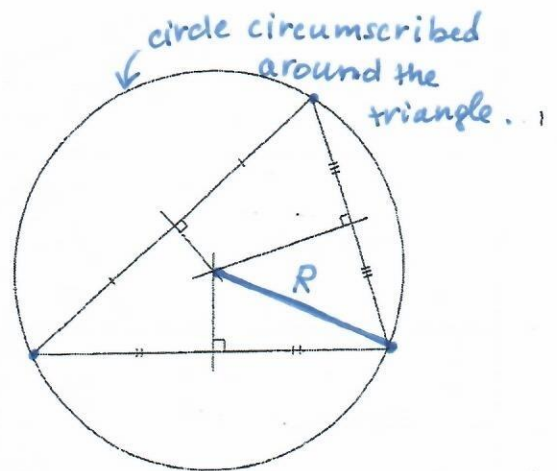


4. The circumcentre

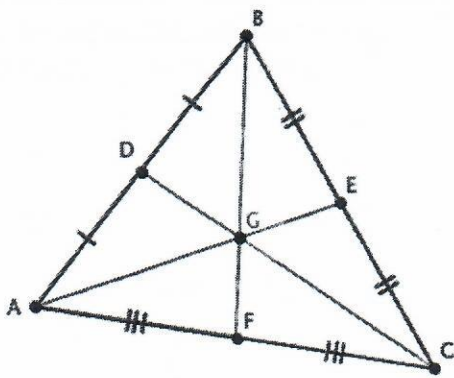
The three perpendicular bisectors (line from mid-point of each side perpendicular to the side) of any triangle are concurrent. The intersection point is called the circumcenter of the triangle, usually denoted by O , and it is the centre of the circumcircle (circle passing through the three vertices of the triangle). The radius R of the circumcircle is called the circumradius of the triangle.

By sine law, we have $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \underline{2R}$ where a ,

b and c are the lengths of the sides opposite to vertices A , B and C . respectively.

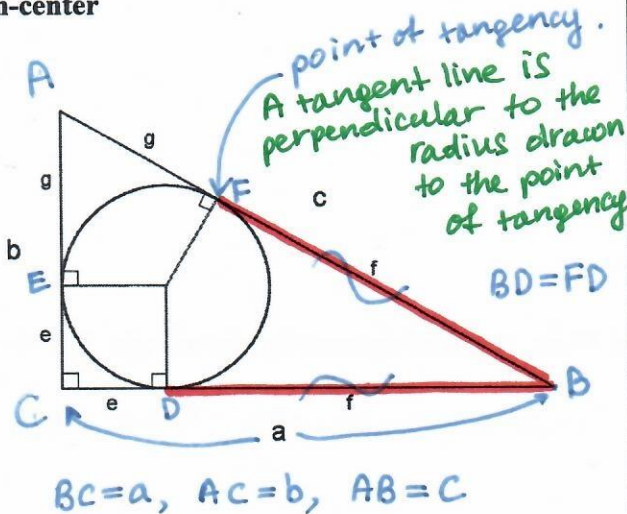


Centroid



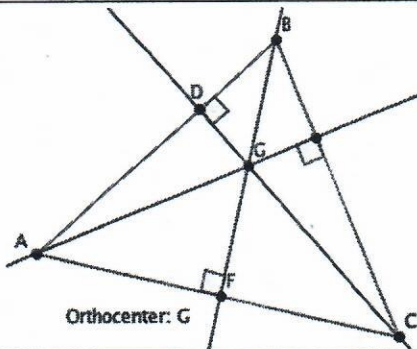
Centroid of a triangle is always located inside the triangle. It is also called the center of mass because the triangular structure acts as if its entire mass was concentrated at that point. That is why one can balance the triangle on a tip of a pencil when the pencil tip is supporting the triangle at the centroid.

In-center



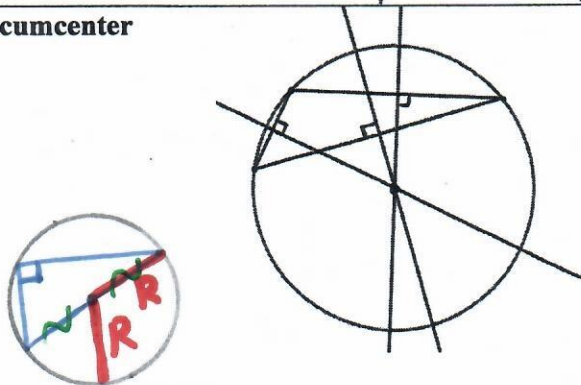
The incenter is the center of the inscribed circle which is the largest circle that can fit inside the triangle. The incenter is always inside the triangle. The sides of the triangle are tangent to the inscribed circle at the points of contact. Those points are called points of tangency. Two line segments tangent to a circle that are drawn from the same external point have the same length.

Orthocenter



As an altitude of a triangle can be outside of the triangle so can orthocenter. In fact if a triangle is obtuse the orthocenter will lie outside the triangle.

Circumcenter



This is the center of the circle circumscribed around the triangle. That is the smallest circle that contains the triangle entirely and passes through the vertices of the triangle. In a particular case of a right triangle the circumcenter is the midpoint of the hypotenuse.

Euler Line

In any triangle, the **centroid**, **circumcenter** and **orthocenter** are always collinear that is they lie on the same straight line, called Euler line in honor of the mathematician Leonhard Euler who discovered it.

on the same line.