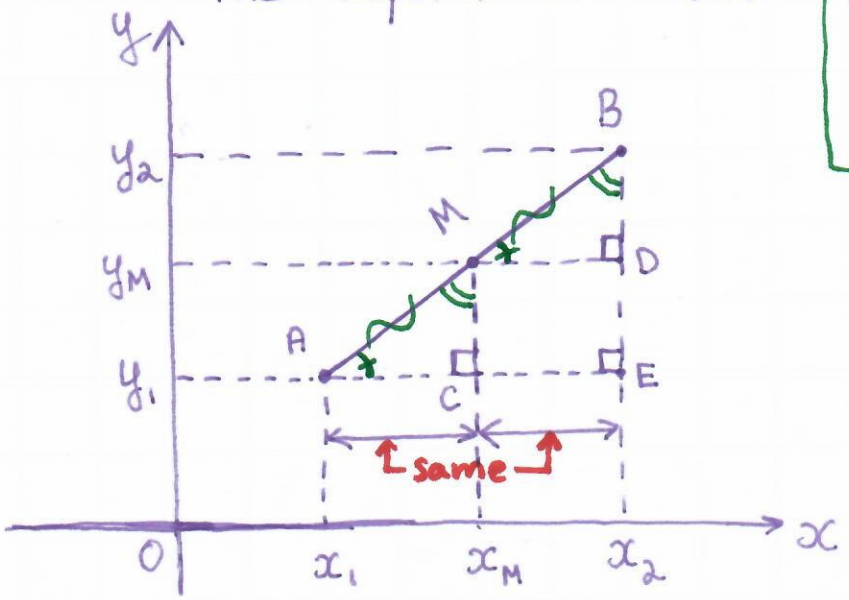
The Midpoint Formula.



Given:
$A(x_1, y_1)$
$B(x_2, y_2)$

M is a midpoint
$AM = MB$
$M_{AB} = (x_M, y_M)$

$\begin{cases} x_M - ? \\ y_M - ? \end{cases}$

$\triangle AMC$, $\triangle MBD$ are both RATs.

$\angle MAC = \angle BMD$ (corresponding angles)

$\angle AMC = \angle MBD$ (corresponding angles)

$\triangle AMC$ is congruent (the same in every respect) to $\triangle MBD$

$\therefore AC = MD = CE$ and $CM = BD = DE$

$$x_M = x_1 + AC = x_1 + \frac{x_2 - x_1}{2} = \frac{2x_1 + x_2 - x_1}{2} = \frac{x_1 + x_2}{2}$$

$$y_M = y_1 + BD = y_1 + \frac{y_2 - y_1}{2} = \frac{2y_1 + y_2 - y_1}{2} = \frac{y_1 + y_2}{2}$$

$$\boxed{M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)}$$

Example 1: Given $A(-1, 2)$, $B(-3, -4)$, find M_{AB}.

$$M_{AB} = \left(\frac{-1 + (-3)}{2}, \frac{2 + (-4)}{2} \right) = \left(\frac{-1-3}{2}, \frac{2-4}{2} \right) = (-2, -1)$$

Example 2: Given $M_{AB} = (-3, 5)$, $A(7, 9)$, B - ?

Let $B(x_B, y_B)$

$$M_{AB} = (-3, 5) = \left(\frac{7 + x_B}{2}, \frac{9 + y_B}{2} \right); \quad \begin{cases} \frac{7 + x_B}{2} = -3 \rightarrow 7 + x_B = -6 \\ \frac{9 + y_B}{2} = 5 \rightarrow 9 + y_B = 10 \end{cases}$$

same

$x_B = -13$, $y_B = 1 \Rightarrow B(-13, 1)$