Factoring.

① To factor means to represent/write as product

Example: $72 = 8 \cdot 9$ (product of two or more quantities)

a factor = a divisor

② Perfect Squares (of Integers)

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169
196, 225, 256, 289, 324, ...

③ Differences of perfect Squares

$4 - 1 = (2)^2 - (1)^2 = 3 = (2+1)(2-1)$
$9 - 4 = (3)^2 - (2)^2 = 5 = (3+2)(3-2)$
$16 - 9 = (4)^2 - (3)^2 = 7 = (4+3)(4-3)$
⋮

Conjecture: $\xleftarrow{\text{expanding}}$
$a^2 - b^2 = (a+b)(a-b)$ ✓ Proof?
$\xrightarrow{\text{factoring}}$

$$a^2 - b^2 = a^2 \underbrace{- ab + ab}_{\text{zero}} - b^2$$

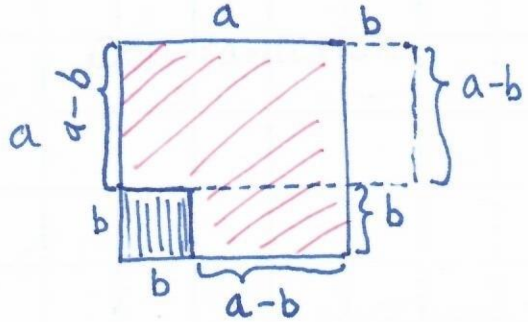
$$= \underbrace{a^2 - ab}_{\text{group 1}} \; \underbrace{+ ab - b^2}_{\text{group 2}}$$

$$= a\underbrace{(a-b)}_{\text{same}} + b\underbrace{(a-b)}_{\text{same.}} = (a-b)(a+b) \checkmark$$

same ⟷ same.

QED :)

$a^2 - b^2 = (a-b)(a+b)$



#1 $n \in \mathbb{N}$ (natural numbers are positive integers)

$\sqrt{n^2+15} \in \mathbb{N}$

$\rightarrow$ let $\sqrt{n^2+15} = a$

Square both sides

$$n^2+15 = a^2 \quad , \quad a \in \mathbb{N} ; \; n \in \mathbb{N}$$

$$15 = a^2 - n^2$$

$$a^2 - n^2 = 15$$

$$\underbrace{(a+n)}_{\in \mathbb{N}} \underbrace{(a-n)}_{\in \mathbb{N}} = 15$$

$a - n > 0$

$\boxed{a > n}$

$$a+n > a-n$$

$a+n$	$a-n$
15	1
5	3

$\begin{cases} a+n = 15 & ① \\ a-n = 1 & ② \end{cases}$

①-②: $2n = 14$

$n = \frac{14}{2} = 7$ ✓

$\begin{cases} a+n = 5 & (*) \\ a-n = 3 & (**) \end{cases}$

$(*) - (**)$: $2n = 2$

$n = 1$ ✓

#8 $15x - 10y = 22$

Lattice point

(x, y); $x, y \in \mathbb{Z}$

$$5(3x - 2y) = 22$$

If $x, y \in \mathbb{Z}$;

$5(3x-2y)$ is an integer multiple of 5

$\boxed{5n, \; n \in \mathbb{Z}}$

But 22 is not an integer multiple of 5.

Contradiction.