Problem Solving w/ Linear Systems

Ex1: Solve for y

$$\begin{cases} x+y+z=25 & ① \\ x+y=19 & ② \\ y+z=18 & ③ \end{cases}$$

method 1': Sub ③ into ①

$18+x=25$, $x=25-18$

$x=7$

Sub into ②:

$7+y=19$, $y=19-7$

$y=12$

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method 2':

② + ③': $2y+x+z=37$ ④

①': $x+y+z=25$

④ − ①': $y=37-25$

$y=12$ ✓

Ex 2':

Prove that $\frac{2n+1}{3n+2}$ is not reducible for $n \in \mathbb{N}$.

- Prove: show evidence, a convincing argument (line of reasoning)
- $\frac{2n+1}{3n+2}$: positive integer over another positive integer.
- not reducible means (both) numerator, denominator are relatively prime. gcd (gcf) (num-r, den-r) = 1
- $n \in \mathbb{N}$: positive integers.

We cannot prove by example!

Because there are infinitely many possibilities. (We cannot list them all!)

Example: $\frac{2(7)+1}{3(7)+2} = \frac{15}{23} = \frac{15(1)}{23(1)}$ ← the only common factor.

We need a general way of reasoning.

If $\frac{2n+1}{3n+2}$ was reducible.

then $2n+1$, $3n+2$ would have a common factor, $d \neq 1$, $d \in \mathbb{N}$

① $2n+1 = da$, $a \in \mathbb{N}$

② $3n+2 = db$, $b \in \mathbb{N}$

But, ... infinitely many cases for n... ?!

Eliminate n:

① × 3: $\boxed{6n+3 = 3da}$ ①'

② × 2: $\boxed{6n+4 = 2db}$ ②

②' − ①' ⇒ $2db - 3da = 1$.

$d(2b-3a) = 1$ $\quad \square \cdot \triangledown = 1$.

↳ $d = 1$, $2b - 3a = 1$.

But $d \neq 1$, ∴ $\frac{2n+1}{3n+2}$ is not reducible.

Ex 1: $\begin{cases} 2x+3y=4 & ① \\ 7x-5y=-17 & ② \end{cases}$

Eliminate y:

① × 5: $10x+15y=20$ ①'

② × 3: $21x-15y=-51$ ②'

①' + ②': $31x=-31$

$x=-\frac{31}{31}$, $x=-1$

Sub ①': $2(-1)+3y=4$

$3y=4+2$

$3y=6$, $y=\frac{6}{3}$, $y=2$

∴ POI: $(-1,2)$

Ex 2: $\begin{cases} x+3y=5 \\ -x+2y=5 \end{cases}$ POI: $(-1,2)$

Ex 3: $\begin{cases} 2x+7y=12 \\ x+4y=7 \end{cases}$ POI: $(-1,2)$

Ex 4: $\begin{cases} 2x+7y=12 \\ 9x+5y=1 \end{cases}$ POI: $(-1,2)$

Ex 5: $\begin{cases} 3x+7y=11 \\ -2x+3y=8 \end{cases}$ POI: $(-1,2)$

(+4, +4 on the first equation; +5, +5 on the second)

Ex 6: $\begin{cases} 3x-y=-5 \\ 6x+13y=20 \end{cases}$ POI: $(-1,2)$