Introduction to Problem Solving.

In problem solving there are often certain tactics (heuristics) that tend to produce results. A heuristic is a method/action that is likely to lead to a solution. Those are derived from extensive experience of many problem solvers. It's a good habit of mind to try to come up with your own heuristics as we go along.

① make a simpler problem ② Establish a pattern.

Example 1a: Evaluate $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$ where n is a natural number. $n \in \mathbb{N}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ positive integers

divisor

Number of Factors	Actual Factors Present	Overall Result
1	$\left(1-\frac{1}{2}\right)$	$\frac{1}{2}$
2	$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}=$	$\frac{1}{3}$
3	$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{4}$	$\frac{1}{4}$
4	$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{4}}{5}$	$\frac{1}{5}$

For n factors, the result is $\frac{1}{n+1}$. Given n-1 factors, $\frac{1}{(n-1)+1}$

Based on the number pattern our conjecture for the answer is: $\frac{1}{n}$

How many factors are there? $n-1$ What is the answer if we have k ~~terms~~ factors? $\frac{1}{k+1}$

Example 1b: Evaluate $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$ where n is a natural number. $\rightarrow \frac{2}{n}$

① $1-\frac{1}{3}=\boxed{\frac{2}{3}}$ ② $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=\frac{2}{3}\cdot\frac{3}{4}=\boxed{\frac{2}{4}}=\frac{1}{2}$ ③ $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)=\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{4}}{5}=\boxed{\frac{2}{5}}$

④ $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)=\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}=\boxed{\frac{2}{6}}=\frac{1}{3}$ Pattern: result $\frac{2}{\text{(denom-r of fraction in the last factor)}}$

HW:

Example 1c: Evaluate $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\ldots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$ where n is a natural number.

Example 1d: Generalizing: Evaluate $\left(1-\frac{1}{a}\right)\left(1-\frac{1}{a+1}\right)\left(1-\frac{1}{a+2}\right)\ldots\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n}\right)$ where n is a natural number.

HW:

Example 2: Evaluate $\frac{8}{4}\cdot\frac{12}{8}\cdot\frac{16}{12}\cdot\ldots\cdot\frac{4n+4}{4n}\cdot\ldots\cdot\frac{2008}{2004}$ where n is a natural number.

Example 3:

Evaluate (a) $1+3+5+7+\ldots+101$

(b) $1+3+5+7+\ldots+2n-1$, where n is a natural number.