Consider a linear system in the form: $\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$

An interesting case occurs if there is an addition/subtraction pattern to coefficients in both equations (not necessarily the same pattern in both)

Example:

$$\begin{cases} x+2y=3 \longrightarrow 1 \xrightarrow{+1} 2 \xrightarrow{+1} 3 \\ -x+3y=7 \longrightarrow -1 \xrightarrow{+4} 3 \xrightarrow{+4} 7 \end{cases}$$

Eliminate x: add equations:

$5y=10$, $\boxed{y=2}$, $x=3-2y$, $x=3-2(2)$ $\boxed{x=-1}$

$\boxed{\text{POI: } (-1,2)}$

Let's try a different example:

$\begin{cases} 2x+4y=6 \\ 5x+6y=7 \end{cases}$ POI: $(-1,2)$ $\begin{cases} 3x+7y=11 \\ x+2y=3 \end{cases}$ POI: $(-1,2)$

Conjecture: For linear systems $\bar{w}$ addition/subtraction pattern in the coefficients, the POI: $(-1,2)$

Proof:

$$\begin{cases} ax+(a+h)y=a+2h & ① \\ dx+(d+k)y=d+2k & ② \end{cases}$$

Eliminate x:

①$\times d$: $dax+d(a+h)y=d(a+2h)$ ③

②$\times a$: $adx+a(d+k)y=a(d+2k)$ ④

③$-$④: $d(a+h)y-a(d+k)y=d(a+2h)-a(d+2k)$

$\cancel{ady}+dhy-\cancel{ady}-aky=\cancel{da}+2hd-\cancel{ad}-2ak$

$y(dh-ak)=2(hd-ak)$

assuming $dh-ak\neq 0$, divide through by $dh-ak\neq 0$, and we get $\boxed{y=2}$

Sub $y=2$ into ①:

$ax+(a+h)(2)=a+2h$

$ax+2a+\cancel{2h}=a+\cancel{2h}$

$ax=a-2a$

$ax=-a$

Divide through by $a\neq 0$ $\rightarrow$ $\boxed{x=-1}$

POI: $(-1,2)$