LCM and GCD (Applications of PND)

↳ least common multiple

↳ greatest common divisor/factor.

Example:

(A) $\text{lcm}(72, 48)$

Guess! 144?

$72 = 2^3 \cdot 3^2$

$48 = 2^4 \cdot 3$

48	2
24	2
12	2
6	2
3	3
1	

$\text{lcm}(72, 48) = 2^4 \cdot 3^2 = 16 \cdot 9 = 144$

Summary: If $N_1 = p_1^{e_1} \cdot p_2^{e_2}$

$N_2 = p_1^{a_1} \cdot p_2^{a_2}$

$$\text{lcm}(N_1, N_2) = p_1^{\max(a_1, e_1)} \cdot p_2^{\max(a_2, e_2)}$$

(B) $\gcd(72, 48) = \text{gcf}(72, 48)$

$72 = 2^3 \cdot 3^2$

$48 = 2^4 \cdot 3$

$\gcd(72, 48) = 2^3 \cdot 3^1 = 24$

Summary: If $N_1 = p_1^{e_1} \cdot p_2^{e_2}$

$N_2 = p_1^{a_1} \cdot p_2^{a_2}$

$$\gcd(N_1, N_2) = \underbrace{p_1^{\min(e_1, a_1)} \cdot p_2^{\min(a_2, e_2)}}_{\text{primes } N_1, N_2 \text{ have in common}}$$

$72 \cdot 48 = 3456$

$\text{lcm}(72, 48) \cdot \gcd(72, 48) = (144)(24) = 3456$

In general $\text{lcm}(a, b) \cdot \gcd(a, b) = ab$. Proof?