Determine all numbers divisible by 30 that have exactly 30 divisors.

$$\begin{array}{c} 30 \\ / \ \backslash \\ 6 \times 5 \\ / \backslash \quad | \\ 2 \times 3 \times 5 \end{array}$$

$30 = 2 \times 3 \times 5$

A number divisible by 30 will have a tentative prime factorization of

$N = 2^a \cdot 3^b \cdot 5^c \cdot \underline{\ \ ?\ \ }$

$t(N) = 30 = 2 \cdot 3 \cdot 5$ ← needs to have at least 3 factors as there are at least 3 primes with exponents in factorization of N.

(30 cannot be written as a product of more than 3 numbers different from 1)

Then $N = 2^a \cdot 3^b \cdot 5^c$ ← exactly 3 primes in decomposition

$t(N) = (a+1)(b+1)(c+1) = 2 \cdot 3 \cdot 5$

$$\left.\begin{array}{l} a+1=2,\ a=1 \\ b+1=3,\ b=2 \\ c+1=5,\ c=4 \end{array}\right\}$$ these exponents can occur on primes in any particular arrangement ("primes do not own exponents")

$2^1 \cdot 3^2 \cdot 5^4 \qquad 2^1 \cdot 3^4 \cdot 5^2$

$2^2 \cdot 3^1 \cdot 5^4 \qquad 2^2 \cdot 3^4 \cdot 5^1$

$2^4 \cdot 3^1 \cdot 5^2 \qquad 2^4 \cdot 3^2 \cdot 5^1$

The numbers are: 11250, 4050, 7500, 1620, 1200, 720.