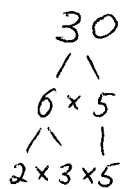


Determine all numbers divisible by 30 that have exactly 30 divisors.



$$30 = 2 \times 3 \times 5$$

A number divisible by 30 will have a tentative prime factorization of

$$N = 2^a \cdot 3^b \cdot 5^c \cdot \underline{\quad ? \quad}$$

$\tau(N) = 30 = 2 \cdot 3 \cdot 5 \leftarrow$  needs to have at least 3 factors as there are at least 3 primes with exponents in factorization of  $N$ .

(30 cannot be written as a product of more than 3 numbers different from 1)

Then  $N = 2^a \cdot 3^b \cdot 5^c \leftarrow$  exactly 3 primes in decomposition

$$\tau(N) = (a+1)(b+1)(c+1) = 2 \cdot 3 \cdot 5$$

$\left. \begin{array}{l} a+1=2, a=1 \\ b+1=3, b=2 \\ c+1=5, c=4 \end{array} \right\}$  these exponents can occur on primes in any particular arrangement ("primes do not own exponents")

$$2^1 \cdot 3^2 \cdot 5^4 \quad 2^1 \cdot 3^4 \cdot 5^2$$

$$2^2 \cdot 3^1 \cdot 5^4 \quad 2^2 \cdot 3^4 \cdot 5^1$$

$$2^4 \cdot 3^1 \cdot 5^2 \quad 2^4 \cdot 3^2 \cdot 5^1$$

The numbers are: 11250, 4050, 7500, 1620, 1200, 720.