

1000. Linear Equations and Linear Systems - Problem Solving - 1

① The numbers A , B , and C are integers such that $A + 3 = B + C$. A certain number is 1 more than $2A$, is 2 more than $3B$, and 3 more than $4C$. What is this number?

② Prove that any two consecutive odd integers are relatively prime.

③ Three straight lines $y = kx + b$, $y = 2kx + 2b$ and $y = bx + k$ are all different and intersect at one point. What possible coordinates does POI have?

④ For what value of a does the system

$$\begin{cases} 2x - ay = 3 \\ 6x - 9y = 9 \end{cases}$$

have an infinite set of solutions?

⑤ For what value of a does the system

$$\begin{cases} 4x + 3y = 12 \\ 2x + ay = 7 \end{cases}$$

have no solutions?

⑥ Suppose you are solving the system $\begin{cases} y = 2x + m \\ 3x - y = n \end{cases}$, $m, n \in \mathbb{Z}$
 Could this system have solutions in all four quadrants?
 Justify your answer.

⑦ The speed of a helicopter exceeds the speed of a car by 70 km/h and the ratio of their speeds is 15:8. Find each speed.

⑧ The point $(a, 2)$ is the point of intersection of the lines with equations $y = 2x - 4$ and $y = x + k$. Determine the value of k .

⑨ Solve the linear system:

$$\begin{cases} 3x + 5y + 2z = 19 \\ 4x - y - 3z = 8 \\ 2x - 2y - 3z = 1 \end{cases}$$

Answers

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② let $2n+1, 2n+3..$

③ (1, 0)

④ $a=3$

⑤ $a=\frac{3}{2}$

⑥ yes, the system can have solutions in all four quadrants (depending m, n); $m, n \in \mathbb{Z}$

⑦ $V_{\text{car}} = 80 \text{ km/h}$

$$V_{\text{helicopter}} = 150 \text{ km/h}$$

⑧ $k = -1$

⑨ $(x, y, z) = (2, 3, -1)$