Oct 4/2019

#10 $p \Rightarrow$ prime
$p \neq 2$ } p is an odd prime

- Since p is odd, p^2 is also an odd number.

Aside: $x \in \mathbb{Z}$

$$(2x+1)^2 = (2x+1)(2x+1)$$
$$= 4x^2+4x+1$$
$$= 4x(x+1)+1$$
$$= 2m+1 \leftarrow \text{yet another odd number.}$$
$$m = 2x(x+1),\ m \in \mathbb{Z}$$

- p^2+13 is a sum of two odd numbers and is, therefore, an even number.

Aside:

let $2x+1$ rep. the first odd number.
let $2a+1$ rep the 2nd odd number.

$$(2x+1)+(2a+1) = 2x+2a+1+1 \qquad x \in \mathbb{Z}$$
$$= 2x+2a+2 \qquad a \in \mathbb{Z}$$
$$= 2(x+a+1)$$
$$= 2m,\ m = x+a+1,\ m \in \mathbb{Z}$$

integer multiple of 2 is an even number.

Therefore

- p^2+13 is an even number greater than 2 which is a composite number.

#11 $\underbrace{p \in \mathbb{N},\ q \in \mathbb{N}}_{\text{prime}}$

pq:

Case 1: $p=q=2$; $pq=4$ ← p, q identical

Case 2: $\begin{cases} p=2 \\ q=3 \end{cases}$; $pq=6$ ← p, q distinct,

Divisor Counting - Introduction.

divisor = factor

if m is a divisor of n, m goes into n evenly (without remainder), n is an integer multiple of m.

$$n = mk, \ k \in \mathbb{Z}; \ m, n \in \mathbb{Z} \qquad |m| \leq |n|$$

How many positive integer divisor does a positive integer have?

Example 1: $N = 72$. List: $\underbrace{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}_{\text{12 divisors.}}$

$$\begin{array}{r|l} 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$72 = 2^3 \cdot 3^2$

Any divisor of 72 will only have 2s and/or 3s in its prime number decomp. (PND).

Any divisor of 72 is of the form

$$2^a \cdot 3^b \qquad \underbrace{a \leq 3, \quad b \leq 2}_{\text{non-negative integers, } \mathbb{W}}$$

$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$

a: 0, 1, 2, 3 ← 4 possibilities

b: 0, 1, 2 ← 3 possibilities.

$$\underset{\text{"t of 72"}}{t(72)} = \underset{(3+1)}{(4)}\underset{(2+1)}{(3)} = 12 \leftarrow \text{Principle of multiplying possibilities.}$$

b \ a	0	1	2	3
0	$2^0 \cdot 3^0$	$2^1 \cdot 3^0$	$2^2 \cdot 3^0$	$2^3 \cdot 3^0$
1	$2^0 \cdot 3^1$	$2^1 \cdot 3^1$	$2^2 \cdot 3^1$	$2^3 \cdot 3^1$
2	$2^0 \cdot 3^2$	$2^1 \cdot 3^2$	$2^2 \cdot 3^2$	$2^3 \cdot 3^2$

3 groups of 4 (or 4 groups of 3)

All the divisors of 72.

What about $N = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_k^{e_k}$, $t(n)$ - ?

Example 2: How many positive integer divisors (including 1 and the number itself) does 200 have?

① PND of $N=200$

$$\begin{array}{r|l} 200 & 2 \\ 100 & 2 \\ 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

$$200 = 2^3 \cdot 5^2$$

② Any factor/divisor of 200 is of the form $2^a \cdot 5^b$

a: 0, 1, 2, 3 $\leftarrow$ 4 possibilities

b: 0, 1, 2 $\leftarrow$ 3 possibilities.

③ multiplying the possibilities

$$t(200) = (4)(3)$$
$$= (3+1)(2+1) = 12$$

$\boxed{\text{Remark:}}$ As Ex1, Ex2 show it is the exponents that matter as far as divisor counting is concerned.

Example:

$2^5 \cdot 7^9$ has the same number of divisor $3^5 \cdot 5^9$ or $7^5 \cdot 11^9$

If $N = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_k^{e_k}$

$$t(N) = (e_1+1)(e_2+1) \cdot \ldots \cdot (e_k+1).$$