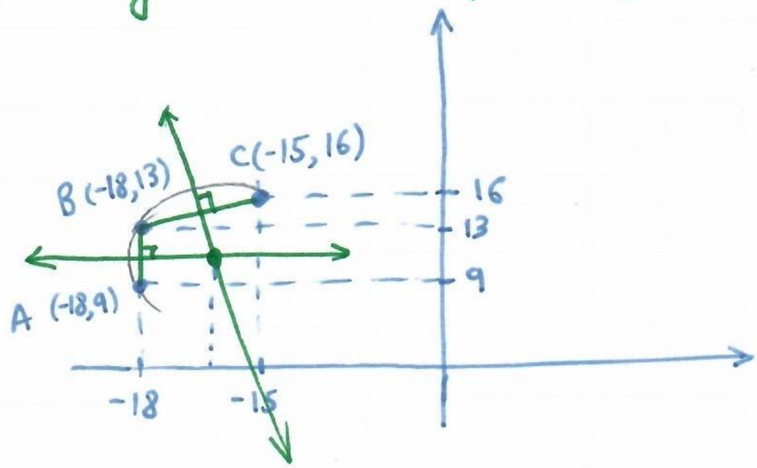
#20 3 non-collinear points specify a circle.

AG: Think graphically, solve analytically!

$-18 \le x \le -15$

$9 \le y \le 16$



$y_c = \left(\frac{9+13}{2}\right) = \frac{22}{2} = 11$, $x_c - ?$

$M_{BC} = \left(\frac{-18+(-15)}{2}, \frac{13+16}{2}\right) = \left(-\frac{33}{2}, \frac{29}{2}\right)$

$m_{BC} = \frac{16-13}{-15-(-18)} = \frac{3}{3} = 1$, $m_{\underline{BC}} = -1$

$y = -x + b$ sub $\left(-\frac{33}{2}, \frac{29}{2}\right)$

$\frac{29}{2} = -\left(\frac{-33}{2}\right) + b$, $b = \frac{29}{2} - \frac{33}{2}$, $b = \frac{-4}{2} = -2$

$$\begin{cases} y = -x - 2 & (1) \\ y = 11 & (2) \end{cases}$$

comparison:

$-x - 2 = 11$

$-x = 13$

$x = -13$

centre $(-13, 11) = D$

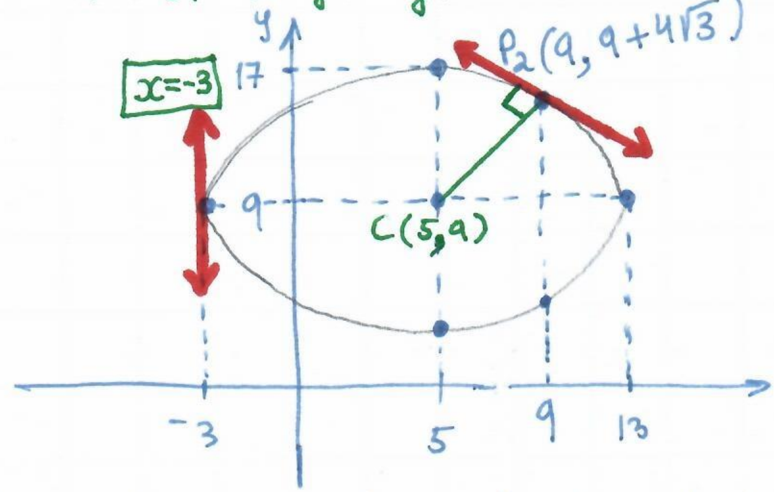
$d_{DC} = \sqrt{(-13-(-15))^2 + (11-(16))^2} = \sqrt{4 + (-5)^2} = \sqrt{4+25}$
$= \sqrt{29}$.

$(x+13)^2 + (y-11)^2 = 29$.

#7 (c) $(x-5)^2+(y-9)^2=64 \leftarrow$ Centre: $(5,9)$; $r=8$

$P_1(-3,9) \leftarrow$ Point of Tangency.

$$\boxed{\begin{array}{c} -3 \le x \le 13 \\ 1 \le y \le 17 \end{array}}$$



Sub $x=9$ into the equation of the circle

$(9-5)^2+(y-9)^2=64$

$16+(y-9)^2=64$, $\quad (y-9)^2=64-16$

$(y-9)^2=48$

$y-9=\pm\sqrt{48}$

$y-9=\pm\sqrt{(16)(3)}=\pm\sqrt{16}\sqrt{3}$

$y-9=\pm 4\sqrt{3}$

$y=9\pm 4\sqrt{3}$

$m_{CP_2}=\dfrac{\cancel{9}+4\sqrt{3}-\cancel{9}}{9-5}=\dfrac{4\sqrt{3}}{4}=\sqrt{3}$; $\qquad m_{\underline{\perp P_2}}=-\dfrac{1}{\sqrt{3}}$

$y=-\dfrac{1}{\sqrt{3}}x+b \quad$ use $(9, 9+4\sqrt{3})$

$9+4\sqrt{3}=-\dfrac{1}{\sqrt{3}}(9)+b$, $\quad b=9+4\sqrt{3}+\dfrac{9}{\sqrt{3}}$; $b=9+4\sqrt{3}+\dfrac{(3)(3)}{\sqrt{3}}$

$b=9+4\sqrt{3}+\dfrac{(3)(\sqrt{3})^{\cancel{2}}}{\cancel{\sqrt{3}}}$

$b=9+4\sqrt{3}+3\sqrt{3}$; $\quad b=9+7\sqrt{3}$

$y=-\dfrac{1}{\sqrt{3}}x+9+7\sqrt{3}$