

A bathtub, with two taps, can be filled in 20 minutes using only the cold water tap. It can be filled in 30 minutes using only the hot water tap. The flow of each tap is not changed when both taps are turned on. It takes 24 min to drain the full tub. Starting with an empty tub and the drain plug in place, the cold water is turned on. Five minutes later the hot water is also turned on, and five minutes after that the drain plug is removed. How many additional minutes, after the plug is removed, would it take to fill the tub?

Solution: Using the cold water tap, $\frac{1}{20}$ th of the tub is filled per minute. After 5 min., the tub is $5\left(\frac{1}{20}\right) = \frac{5}{20} = \frac{1}{4}$ full. Both cold and hot water taps, we fill the tub at a rate of $\left(\frac{1}{20} + \frac{1}{30}\right) = \frac{-3+2}{60} = -\frac{5}{60} = \frac{1}{12}$ min. After 5 min, $\frac{1}{4} + 5\left(\frac{1}{12}\right) = -\frac{1}{4} + \frac{5}{12} = \frac{-3+5}{12} = \frac{2}{12} = \frac{1}{6}$ full. It remains to fill $\frac{1}{6}$ of the tub.

The $\frac{1}{3}$ of the tub is filled at
a rate of $\left(\frac{1}{20} + \frac{1}{30} - \frac{1}{24}\right) =$
 $= \left(\frac{6}{120} + \frac{4}{120} - \frac{5}{120}\right) = \frac{5}{120} = \frac{1}{24} \text{ tub/min.}$

time = $\frac{\frac{1}{3}}{\frac{1}{24}} = \frac{1}{3} \times \frac{24}{1} = \frac{24}{3} = 8 \text{ min.}$