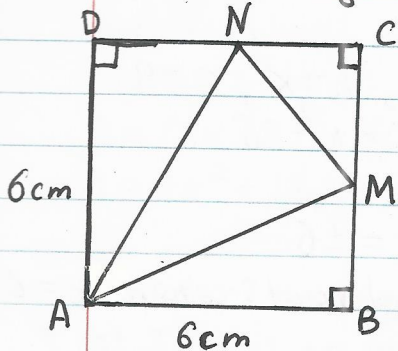


A square ABCD has sides of length 6cm.

M and N are points on sides BC and DC such that the areas of $\triangle ABM$, $\triangle MCN$, and $\triangle ADN$ are all equal. Find the lengths of BM and DN.



Solution: $A_{\triangle ABM} = A_{\triangle MCN} = A_{\triangle ADN}$

$$\frac{6(DN)}{2} = \frac{(NC)(CM)}{2} = \frac{(BM)(6)}{2} \text{ so } 6(DN) = (NC)(CM) = 6BM$$

so $DN = BM$. Let $DN = BM = x$

then

$$6x = (6-x)(6-x) = 6x, \quad 6x = (6-x)^2$$

$$6x = 36 - 12x + x^2$$

$$x^2 - 18x + 36 = 0$$

$$x_{1,2} = \frac{18 \pm \sqrt{18^2 - 4(1)(36)}}{2} = \frac{18 \pm \sqrt{324 - 144}}{2} = \frac{18 \pm \sqrt{180}}{2}$$

$$x_{1,2} = \frac{18 \pm 6\sqrt{5}}{2} = 9 \pm 3\sqrt{5}$$

But $9 + 3\sqrt{5}$ can not work so

$$BM = DN = x = 9 - 3\sqrt{5} \approx 2.29 \text{ cm} \checkmark$$