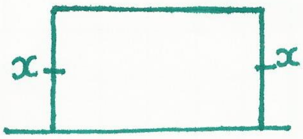
Prove that when fencing 3 sides of a rectangular lot with a fixed perimeter (amount of fencing), the maximum are will occur when length is double the width.

let P represent the perimeter of fencing

① Problem has a geometric context: draw a diagram.



let x represent the width, then

let $P-2x$ replesent the length.

② Set up the area (dep. var) relation

$$A = lw$$

$$A = x(P-2x)$$ now expand and rearrange in standard form

$$A = Px - 2x^2$$

$$A = -2x^2 + Px$$

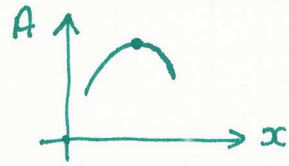
$$A = -2\left(x^2 - \frac{P}{2}x\right)$$ and we complete the square

③ Complete the square

$$A = -2\left(x^2 - \frac{P}{2}x + \frac{P^2}{16} - \frac{P^2}{16}\right) \qquad \left(\frac{-P}{2} \cdot \frac{1}{2}\right)^2$$

$$A = -2\left(x^2 - \frac{P}{2}x + \frac{P^2}{16}\right) + \frac{P^2}{8}$$

$$A = -2\left(x - \frac{P}{4}\right)^2 + \frac{P^2}{8} \longrightarrow$$



④ State:

The max value of A is $\frac{P^2}{8}$

when $x = \frac{P}{4}$ (width)

$$\text{length} = P - 2\left(\frac{P}{4}\right)$$

$$= P - \frac{P}{2}$$

$$= \frac{P}{2}$$

⑤ Conclusion:

$$\frac{\text{length}}{\text{width}} = \frac{\frac{P}{2}}{\frac{P}{4}} = \frac{P}{2} \cdot \frac{4}{P} = 2 \rightarrow \text{length} = 2(\text{width}).$$