

Simplify

$$(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$$

Consider the conjugate of the first binomial: $(2-1)=1$

Multiply the original expression by $(2-1)=1$. The value remains unchanged

$$(2-1)(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$$

$$=(2^2-1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$$

$$=(2^4-1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$$

$$=(2^8-1)(2^8+1)(2^{16}+1)(2^{32}+1)$$

$$=(2^{16}-1)(2^{16}+1)(2^{32}+1)$$

$$=(2^{32}-1)(2^{32}+1)$$

$$=2^{64}-1$$