

Translate from English into algebra:

$$a^2 - b^2 = p; \quad a, b \in \mathbb{Z} \text{ and } p \text{ is prime.}$$

↳ since $a^2 - b^2 = (a-b)(a+b)$

↳ $(a-b)(a+b) = p$

then

$$\begin{cases} a-b=1 \textcircled{1} \\ a+b=p \textcircled{2} \end{cases} \text{ as only factors of } p \text{ are } 1 \text{ and } p \text{ itself}$$

$$\textcircled{1} + \textcircled{2}:$$

$$2a = p+1, \quad a = \frac{p+1}{2}$$

$$\textcircled{2} - \textcircled{1}: \quad 2b = p-1, \quad b = \frac{p-1}{2}$$