

$$y = ax^2 + bx + c$$

x	y	1st diff.	2nd. Diff
-4	$16a - 4b + c$	$-7a + b$	2a
-3	$9a - 3b + c$	$-5a + b$	
-2	$4a - 2b + c$	$-3a + b$	2a
-1	$a - b + c$	$-a + b$	2a
0	c		2a
1	$a + b + c$	$a + b$	2a
2	$4a + 2b + c$	$3a + b$	2a
3	$9a + 3b + c$	$5a + b$	2a
4	$16a + 4b + c$	$7a + b$	2a
5	$25a + 5b + c$	$9a + b$	2a

Given  $\Delta x = 1$ , 2nd diff =  $2a = \text{const}$ ,  $a \neq 0$

We know that the sign of a indicates the direction of opening of the parabola.

Indeed, if  $a > 0$ , parabola opens up. If  $a < 0$  parabola opens down.

We see that non-zero constant 2nd differences will have the same sign as the leading coefficient, a.

Therefore, if non-zero constant 2nd diff  $> 0$  parabola opens up. If non-zero constant 2nd diff  $< 0$ , parabola opens down.

Example 1:  $y = x^2$

x	y	1st diff	2nd diff
-3	9	-5	2
-2	4	-3	
-1	1	-1	2
0	0	1	2
1	1	3	2
2	4	5	2
3	9		2

Since 2nd diff  $> 0$ , parabola opens up.

Example 2:  $y = -x^2$

x	y	1st diff	2nd diff
-3	-9	5	-2
-2	-4	3	
-1	-1	1	-2
0	0	-1	-2
1	-1	-3	-2
2	-4	-5	-2
3	-9		-2