

let a, b represent the two numbers

$$\text{lcm}(a, b) = 600; \quad \frac{a}{b} = \frac{3}{5}$$

How do we start? To get more information about a, b we could consider the PND of 600.

600		2
300		2
150		2
75		3
25		5
5		5

①

$$600 = 2^3 \cdot 3 \cdot 5^2$$

At least one of a, b must have 2's in PND

Yet $\frac{a}{b} = \frac{3}{5}$ ← no two's

So 2's must have cancelled out meaning there are as many 2's in a as there are in b .

In other words, if $a = 2^{e_1} \cdot 3^{e_2} \cdot 5^{e_3}$
 $b = 2^{m_1} \cdot 3^{m_2} \cdot 5^{m_3}$

the exponents on 2 are equal: $e_1 = m_1$, so

$$\max(e_1, m_1) = e_1 = m_1 = 3$$

So $a = 2^3 \cdot 3^{e_2} \cdot 5^{e_3}$ and $b = 2^3 \cdot 3^{m_2} \cdot 5^{m_3}$

Also, $5a = 3b$, which means b is a multiple of 5 and a is divisible (is a multiple of) by 3

a	b
$2^3 \cdot 3 \cdot 5$	$2^3 \cdot 5^2$

One of a, b should have 5^2 in its PND (see lcm): $\max(e_3, m_3) = 2$

Also b should have one more 5 to produce $\frac{a}{b} = \frac{3}{5}$ so 5 occurs to exponents of 1 and 2

$$\therefore a + b = 120 + 200$$