

Solution:

$$\text{let } m^2 + 6m + 28 = n^2, n \in \mathbb{Z}.$$

Rewrite as

$$m^2 + 6m + 9 + 19 = n^2$$

$$(m+3)^2 + 19 = n^2, n^2 - (m+3)^2 = 19$$

$(n-m-3)(n+m+3) = 19$. There are four cases/possibilities.

$$\textcircled{1} \begin{cases} n-m-3=1 \\ n+m+3=19 \end{cases}, \begin{cases} n-m=4 \\ n+m=16 \end{cases}, \begin{cases} n=10 \\ m=6 \end{cases}$$

$$\textcircled{2} \begin{cases} n-m-3=19 \\ n+m+3=1 \end{cases}, \begin{cases} n-m=22 \\ n+m=-2 \end{cases}, \begin{cases} n=10 \\ m=-12 \end{cases}$$

$$\textcircled{3} \begin{cases} n-m-3=-1 \\ n+m+3=-19 \end{cases}, \begin{cases} n-m=2 \\ n+m=-22 \end{cases}, \begin{cases} n=-10 \\ m=-12 \end{cases}$$

$$\textcircled{4} \begin{cases} n-m-3=-19 \\ n+m+3=-1 \end{cases}, \begin{cases} n-m=-16 \\ n+m=-4 \end{cases}, \begin{cases} n=-10 \\ m=6 \end{cases}$$

Answer: $m=6$ or $m=-12$.