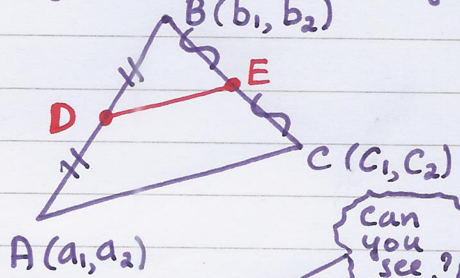
Verifying Properties of Geometric Figures.

Midsegment in a Triangle.



Consider a midsegment DE.

$\boxed{\text{Claim:}}$

① $DE \parallel AC$

② $DE = \frac{1}{2} AC$

Can you see how?

$$D = \left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}\right); \quad E = M_{BC} = \left(\frac{b_1+c_1}{2}, \frac{b_2+c_2}{2}\right)$$

$$m_{DE} = \frac{\frac{b_2+c_2}{2} - \frac{a_2+b_2}{2}}{\frac{b_1+c_1}{2} - \frac{a_1+b_1}{2}} = \frac{b_2+c_2-a_2-b_2}{b_1+c_1-a_1-b_1} = \frac{c_2-a_2}{c_1-a_1}$$

$m_{AC} = \dfrac{c_2-a_2}{c_1-a_1}$; Since $m_{DE} = m_{AC}$, $DE \parallel AC$ ✓

Now to the 2nd part of the claim...

$$d_{DE} = DE = \sqrt{\left(\frac{b_1+c_1}{2} - \frac{a_1+b_1}{2}\right)^2 + \left(\frac{b_2+c_2}{2} - \frac{a_2+b_2}{2}\right)^2}$$

$$d_{DE} = \sqrt{\left(\frac{b_1+c_1-a_1-b_1}{2}\right)^2 + \left(\frac{b_2+c_2-a_2-b_2}{2}\right)^2}$$

$$d_{DE} = \sqrt{\frac{(c_1-a_1)^2}{4} + \frac{(c_2-a_2)^2}{4}} = \frac{\sqrt{(c_1-a_1)^2+(c_2-a_2)^2}}{\sqrt{4}} = \ldots$$

$$d_{DE} = \frac{\sqrt{(c_1-a_1)^2+(c_2-a_2)^2}}{2}.$$

Now $d_{AC} = AC = \sqrt{(c_1-a_1)^2 + (c_2-a_2)^2}$

We have $d_{DE} = \frac{1}{2} d_{AC}$ or $DE = \frac{1}{2} AC$.