Linear Systems with Literal Coefficients.

rectangular table of numbers.

Coefficient matrix: $\begin{bmatrix} a & b \\ d & e \end{bmatrix}$

det = ae − bd

Given a linear system:

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

Solve for x and y.

Solution: The first question is whether we need to use substitution or elimination.

We use elimination to postpone dealing with fractions for a bit. Let's eliminate x.

$$\begin{cases} ax + by = c & (1) \\ dx + ey = f & (2) \end{cases}$$

(1)×d: $dax + dby = dc$ (3)

(2)×a: $adx + aey = af$ (4)

(3)−(4): $dby - aey = dc - af$

Factor out y: $y(db - ae) = dc - af$

$$y = \frac{dc - af}{db - ae} = \frac{-1(dc - af)}{-1(db - ae)}$$

$$\boxed{y = \frac{af - dc}{ae - db}}$$

$$\boxed{x = \frac{ce - bf}{ae - db}}$$

Sub into (1) and solve for x:

$$ax + b\left(\frac{af - dc}{ae - db}\right) = c$$

$$ax + \frac{baf - bdc}{ae - db} = c$$

$$ax = c - \frac{baf - bdc}{ae - db}$$

brackets around numerator.

$$ax = \frac{c(ae - db)}{ae - db} - \frac{(baf - bdc)}{ae - db}$$

$$ax = \frac{cae - \cancel{cdb} - baf + \cancel{bdc}}{ae - db}$$

$$ax = \frac{cae - baf}{ae - db} \rightarrow ax = \frac{a(ce - bf)}{ae - db}$$. Divide through by a.

Therefore the POI is: $\left(\frac{ce - bf}{ae - db}, \frac{af - dc}{ae - db}\right)$ as long as $ae - db \neq 0$

If $ae - bd = 0$, then either there are no solutions or there are infinitely many solutions.

If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, one equation is a multiple of the other, infinitely many solutions

det. = ae − bd = 0

Example:

$$\begin{cases} 2x + 3y = 5 & (1) \\ 4x + 6y = 10 & (2) \end{cases} \rightarrow \frac{2}{4} = \frac{3}{6} = \frac{5}{10}$$ which means (1)×2 = (2)

det = (2)(6) − (4)(3) = 0

If $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$, then one equation contradicts the other; e.g. $\begin{cases} 3x + 4y = 7 \\ 6x + 8y = 10 \end{cases}$

det = ae − bd = 0 ↳ no solutions

Practice

1. Given that the system of equations $\begin{matrix} 3x + my = 7 \\ 2x + ny = 4 \end{matrix}$ has no solutions, where m, n are integers between – 10 and 10 inclusive, find the values of m and n.

2. Determine the values of k such that the system of equations $\begin{matrix} kx - y = -\frac{1}{3} \\ 3y = 1 - 6x \end{matrix}$ has a unique solution, no solution and infinitely many solutions respectively.