Find the number of distinct positive divisors of $(30)^4$ excluding 1 and $(30)^4$

Solution:

$(30)^4 = (2 \cdot 3 \cdot 5)^4 = 2^4 \cdot 3^4 \cdot 5^4$

$t(30^4) = (4+1)^3 = 125$, taking out 1 and (30^4) itself leaves $125 - 2 = 123$ positive divisors

How many positive cubes divide $3! \cdot 5! \cdot 7!$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Solution:

$3! \cdot 5! \cdot 7! = (3 \cdot 2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$

$= 2^8 \cdot 3^4 \cdot 5^2 \cdot 7^1$

A perfect cube that divides $3! \cdot 5! \cdot 7!$ must be of the form $2^a \cdot 3^b \cdot 5^c \cdot 7^d$ where a, b, c, d are non-negative multiples of 3 that are less than or equal to 8, 4, 2 and 1, respectively.

a: 0, 3, 6

b: 0, 3

c: 0

d: 0

$3 \cdot 2 \cdot 1 \cdot 1 = 6 \Rightarrow$ (E) is the Answer.