

If m and n are different positive integers, and if

$$\frac{\frac{1}{m} - \frac{1}{n}}{1 - \frac{1}{mn}} = 1, \text{ what is the value of } m?$$

Solution: $m > 0, n > 0; m, n \in \mathbb{Z}$

Set the numerator equal to the denominator to get

$$\frac{1}{m} - \frac{1}{n} = 1 - \frac{1}{mn}$$

multiply through by $mn \neq 0$

$$n - m = mn - 1$$

$$n + 1 = mn + m = m(n + 1)$$

$$n + 1 - m(n + 1) = 0$$

$$(n + 1)(1 - m) = 0$$

$$n + 1 = 0 \text{ or } 1 - m = 0$$

Either $n = -1$ (not a positive integer!)
or $m = 1$ (works!)

Answer: $\boxed{m = 1}$