Divisor Counting Problems - 2

Problem 1: If n has exactly 7 positive divisors, how many positive divisors does n^2 have?

Problem 2: How many of the positive divisors of 960 have 6 positive divisors?

We need to consider divisors of 960 that possess a certain property (have 6 positive divisors)

The prime number factorization of 960 is: $960 = 2^6 \cdot 3^1 \cdot 5^1$

Any divisor of 960 has the form: $2^a \cdot 3^b \cdot 5^c$

$2^a \cdot 3^b \cdot 5^c$ needs to have exactly 6 positive divisors.

We consider possible combinations of a, b and c such that

$(a+1)(b+1)(c+1) = 6$

The only ways to get 6 as a product of 3 positive integers (order not-withstanding for now) are

$6 = 1 \cdot 1 \cdot 6 = 1 \cdot 2 \cdot 3$

Then a, b, c must include two 0's and one 5 or one 0, one 1, one 2.

But $0 \leq a \leq 6$, $0 \leq b \leq 1$, $0 \leq c \leq 1$ gives:

$a=5, b=0, c=0 \rightarrow 2^5 \cdot 3^0 \cdot 5^0 = 32$, $t(32) = t(2^5) = 6$ ✓ checks

$a=2, b=1, c=0 \rightarrow 2^2 \cdot 3^1 \cdot 5^0 = 12$, $t(12) = t(2^2 3^1) = 6$ ✓ checks

$a=2, b=0, c=1 \rightarrow 2^2 \cdot 3^0 5^1 = 20$, $t(20) = t(2^2 5^1) = 6$ ✓ checks

Remark: It's a very good idea to check solutions, particularly for more involved problems.