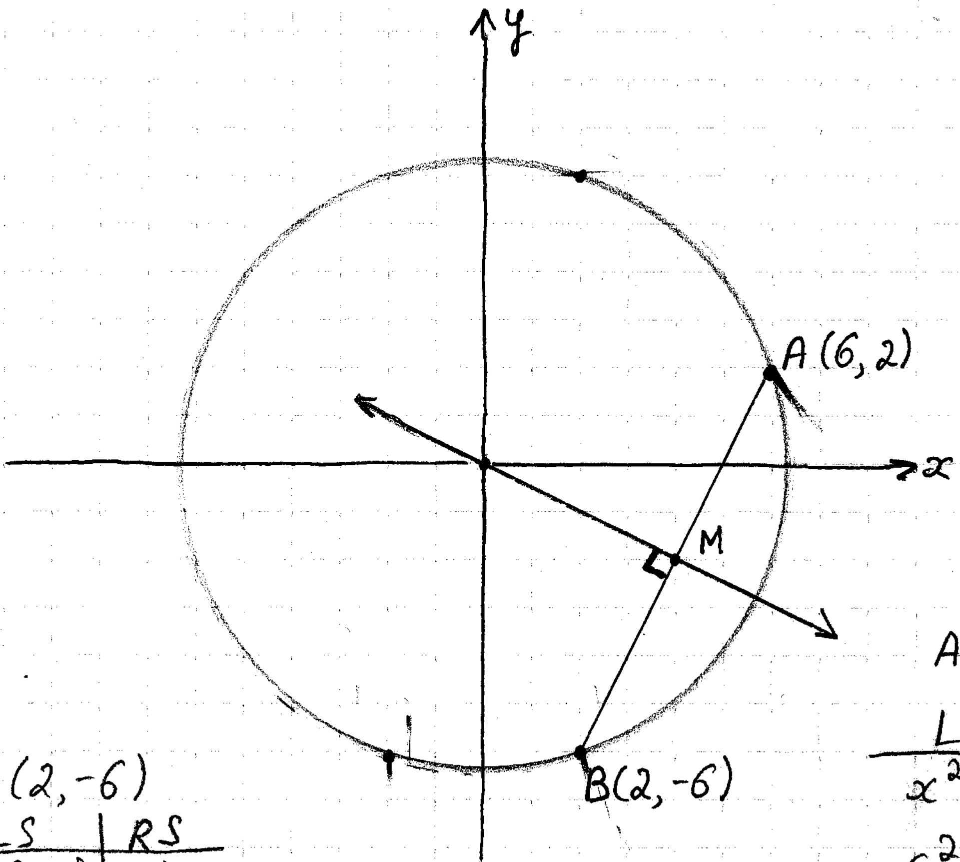
#4 (a) A line segment has endpoints $A(6,2)$; $B(2,-6)$. Show that AB is a chord of the circle $x^2+y^2=40$.

$x^2+y^2=40$ is a circle centered at $(0,0)$ of

radius $r=\sqrt{40}=\sqrt{4\times10}=\sqrt{4}\sqrt{10}=2\sqrt{10}$

Aside: How large is $2\sqrt{10}$? Well, how large is $\sqrt{10}$? 10 is between two perfect squares: 9 and 16
$9<10<16$
$\sqrt{9}<\sqrt{10}<\sqrt{16}$, $3<\sqrt{10}<4$
$6<2\sqrt{10}<8$



If AB is a chord A, B must be points on (circumference of) circle. They must satisfy it's equation. We now check

$A(6,2)$

LS	RS
x^2+y^2	$=40$
6^2+2^2	40
$36+4$	40
40	40 ✓ Check!

$B(2,-6)$

LS	RS
x^2+y^2	$=40$
$2^2+(-6)^2$	40
$4+36$	40
40	40 ✓ Check!

(b) Find the equation of the perpendicular Bisector of the chord AB.

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{6+2}{2},\frac{2-6}{2}\right)=(4,-2)$$

$$m_{AB}=\frac{2-(-6)}{6-2}=\frac{8}{4}=2,\quad m_\perp=-\frac{1}{2},\quad y=-\frac{1}{2}x+b$$

Use $M(4,-2)$ to find b:

$-2=-\frac{1}{2}(4)+b$

$-2=-2+b$

$b=0$ ✓

(c) Show that perp. bisector goes through $(0,0)$.

$\boxed{(0,0)\text{ must satisfy the equation}}$

$y=-\frac{1}{2}x$

LS	RS
0	$-\frac{1}{2}(0)$
0	0 ✓ Check