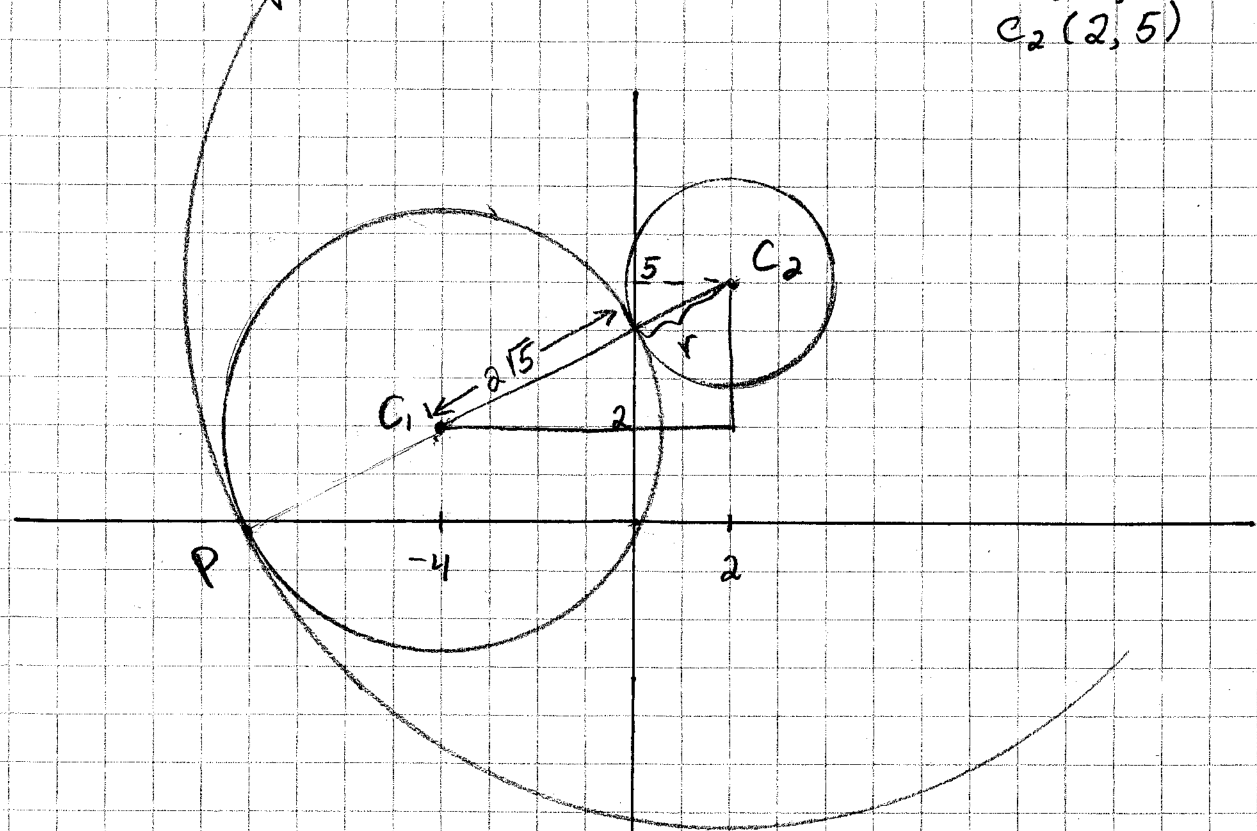
Circle Problems

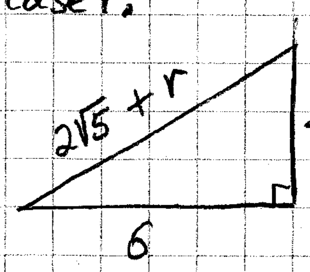
(#3) The circles $(x+4)^2+(y-2)^2=20$ and $(x-2)^2+(y-5)^2=r^2$ just touch at one point. Show with diagrams that two different solutions exist for r and then find these values.

Draw a diagram: centres of circles are: $C_1(-4,2)$
$C_2(2,5)$



The circle $(x+4)^2+(y-2)^2=20$ has a radius of $\sqrt{20}=\sqrt{4\cdot5}=\sqrt{4}\sqrt{5}$
$=2\sqrt{5}\doteq 4.47$

Case 1:



$$(r+2\sqrt{5})^2=6^2+3^2$$

$$r^2+4r\sqrt{5}+20=36+9$$

$$r^2+4r\sqrt{5}+20-45=0$$

$$r^2+4r\sqrt{5}-25=0,\quad a=1,\ b=4\sqrt{5},\ c=-25$$

$$r_{1,2}=\frac{-4\sqrt{5}\pm\sqrt{(4\sqrt{5})^2-4(1)(-25)}}{2(1)}=\frac{-4\sqrt{5}\pm\sqrt{80+100}}{2}$$

$$r_{1,2}=\frac{-4\sqrt{5}\pm\sqrt{180}}{2}=\frac{-4\sqrt{5}\pm\sqrt{36\cdot5}}{2}=\frac{-4\sqrt{5}\pm6\sqrt{5}}{2},\quad r>0$$

$$r=\sqrt{5}$$