Circle Problems: Number 2

Find the equation of a circle with centre at $(k, 2k)$ if the points $(0,7)$ and $(4,3)$ are on the circle.

Solution:

Since $(0,7)$ and $(4,3)$ are on the circle, those points must be the same r units away from the centre, where r is the radius.

$r = \sqrt{(0-k)^2+(7-2k)^2}$; $\quad r = \sqrt{(4-k)^2+(3-2k)^2}$

$r = r$

$$\sqrt{(0-k)^2+(7-2k)^2} = \sqrt{(4-k)^2+(3-2k)^2}$$

Square both sides:

$$(0-k)^2+(7-2k)^2 = (4-k)^2+(3-2k)^2$$

$$(-k)^2+(7-2k)(7-2k) = (4-k)(4-k)+(3-2k)(3-2k)$$

$$k^2+49-14k-14k+4k^2 = 16-4k-4k+k^2+9-6k-6k+4k^2$$

$$49-28k = 25-20k$$

$$49-25 = 28k-20k$$

$$24 = 8k, \quad k = 3$$

The centre then is: $(3, 2(3)) = (3,6)$

The equation becomes (for the circle of course)

$$(x-3)^2+(y-6)^2 = r^2$$

We need r (r^2 actually)

$r^2 = (0-3)^2+(7-2(3))^2 \Rightarrow r^2 = (-3)^2+(1)^2$, $r^2 = 9+1$

$r^2 = 10$

$\therefore$ $\boxed{(x-3)^2+(y-6)^2 = 10}$