

A ship must be unloaded by two cranes in 12 hours. Both cranes worked for 8 hours and then one of them broke. The rest of the work was done by the other crane in 7 hours. How long would each crane take to do the entire job if it was working alone?

Solution:

Entire job, 100% of it, is 1.

As the job is done by both cranes working together in 12 h, their team rate is  $\frac{1 \text{ ship}}{12 \text{ hr}}$ .

Let  $x$  represent the time of the crane that broke. Both cranes worked for 8 h, having done  $\frac{8}{12} = \frac{2}{3}$  of the job. The rest of the job was

done by one crane, the rest of job being  $1 - \frac{2}{3} = \frac{1}{3}$

This was done by the other crane in 7 h.

So the rate of crane that did not break is  $r = \frac{\frac{1}{3}}{7} = \frac{1}{21}$  boat per hr.

First crane does the job in  $x$  hours,  $\frac{1 \text{ ship}}{x \text{ hr}}$ .

Then

$$r_{\text{both}} = \frac{1}{x} + \frac{1}{21} = \frac{1}{12} \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{21} = \frac{1}{12}$$

$$\frac{1}{x} = \frac{1}{12} - \frac{1}{21} \quad \text{lcm} = 84$$

$$\frac{1}{x} = \frac{7}{84} - \frac{4}{84} \quad \Rightarrow \quad \frac{1}{x} = \frac{3}{84} \quad \Rightarrow \quad \frac{1}{x} = \frac{1}{28}$$

$$x = 28 \text{ h}$$

Answer: 28 hr, 21 hr.