The Binomial Theorem (notes) Answers

Date:

Expand and follow the pattern:

$$(x+y)^0 = 1$$

$$(x+y)^1 = x+y$$

$$(x+y)^2 = x^2+2xy+y^2$$

$$\begin{aligned}(x+y)^3 &= (x+y)(x^2+2xy+y^2)\\ &= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3\\ &= x^3+3x^2y+3xy^2+y^3\end{aligned}$$

$$\begin{aligned}(x+y)^4 &= (x+y)(x^3+3x^2y+3xy^2+y^3) \quad \text{"using above expansion"}\\ &= x^4+3x^3y+3x^2y^2+xy^3+x^3y+3x^2y^2+3xy^3+y^4\\ &= x^4+4x^3y+6x^2y^2+4xy^3+y^4\end{aligned}$$

$$\begin{aligned}(x+y)^5 &= (x+y)(x^4+4x^3y+6x^2y^2+4xy^3+y^4)\\ &= \ldots\\ &= x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5\end{aligned}$$

Generalize:

$$\begin{aligned}(x+y)^n &= \sum_{r=0}^{n}\binom{n}{r}x^{n-r}y^r\\ &= \binom{n}{0}x^ny^0+\binom{n}{1}x^{n-1}y^1+\binom{n}{2}x^{n-2}y^2+\ldots+\binom{n}{n}x^0y^n\end{aligned}$$

The Binomial Theorem (notes) Answers

Date:

Ex. Expand $(x-2)^6$.

$$(x-2)^6$$
$$= {}_6C_0\left(x^6\right)(-2)^0 + {}_6C_1\left(x^5\right)(-2)^1 + {}_6C_2\left(x^4\right)(-2)^2 + {}_6C_3\left(x^3\right)(-2)^3$$
$$+ {}_6C_4\left(x^2\right)(-2)^4 + {}_6C_5\left(x^1\right)(-2)^5 + {}_6C_6\left(x^0\right)(-2)^6$$
$$= x^6 + 6(-2)x^5 + (15)(4)x^4 + 20(-8)x^3 + 15(16)x^2 + 6(-32)x + 64$$
$$= x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$$

Ex. Find the first four terms of $\left(2x - \frac{1}{x}\right)^9$.

$$\left(2x - \frac{1}{x}\right)^9$$
$$= \sum_{r=0}^{9} {}_9C_r (2x)^{9-r}\left(-x^{-1}\right)^r$$
$$= \sum_{r=0}^{9} (-1)^r \left({}_9C_r\right)\left(2^{9-r}\right)x^{9-2r}$$
$$= {}_9C_0\left(2^9\right)x^9 - {}_9C_1\left(2^8\right)x^7 + {}_9C_2\left(2^7\right)\left(x^5\right) - {}_9C_3\left(2^6\right)\left(x^3\right) + \ldots$$
$$= 512x^9 - 2304x^7 + 4608x^5 - 5376x^3 + \ldots$$