The Binomial Theorem II – The General Term (notes)

Date:

Ex. In the expansion of $\left(x^2 - \frac{1}{x}\right)^{10}$:

a) how many terms are there?

b) determine the seventh term.

c) the constant term.

d) the coefficient of the term containing x^5.

Solutions

a) Since $r = 0, 1, 2, \ldots, 10$, there are 11 terms.

The Binomial Theorem expansion:

$$\left(x^2 - \frac{1}{x}\right)^{10} = \sum_{r=0}^{10} {}_{10}C_r \left(x^2\right)^{10-r} (-1)^r \left(x^{-1}\right)^r$$

$$= \sum_{r=0}^{10} (-1)^r {}_{10}C_r \left(x^{20-2r}\right)\left(x^{-r}\right)$$

$$= \sum_{r=0}^{10} (-1)^r {}_{10}C_r \left(x^{20-3r}\right)$$

> Notice the subscript, $r+1$. This is because $0 \le r \le n$, but the sequence of terms is $t_1, t_2, t_3, \ldots, t_{n+1}$. For example, the sixth term, t_6 would have $r = 5$.

The General Term: $t_{r+1} = (-1)^r {}_{10}C_r \left(x^{20-3r}\right)$

b) $\therefore t_7 = (-1)^6 {}_{10}C_6 \left(x^{20-3(6)}\right) = 210x^2$

c) x^0?

$20 - 3r = 0$

$3r = 20$

$r = \frac{20}{3}$

But r is a whole number from zero to n. $\therefore$ there is no constant term.

d) $20 - 3r = 5$

$3r = 15$

$r = 5$

$t_6 = (-1)^5 {}_{10}C_5 \left(x^{20-3(5)}\right) = -252x^5$

$\therefore$ the coefficient of x^5 is -252.