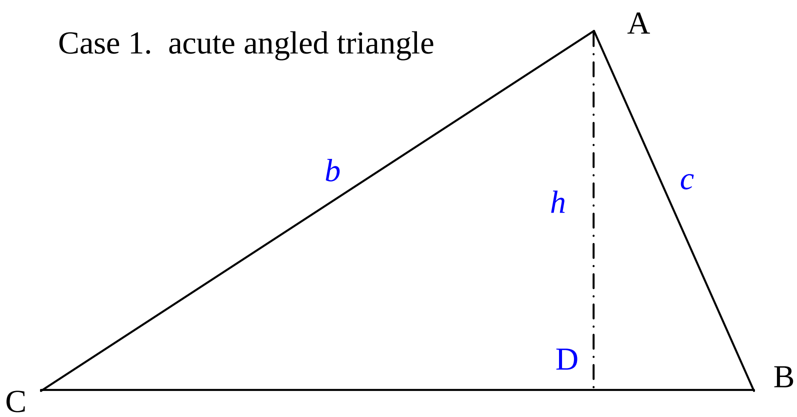
Proving Sine Law & Cosine Law (solutions)

Derive the formula for Sine Law.

Case 1. acute angled triangle



In ΔACD,

$$\sin C = \frac{h}{b}$$
$$h = b\sin C \quad ①$$

In ΔADB,

$$\sin B = \frac{h}{c}$$
$$h = c\sin B \quad ②$$

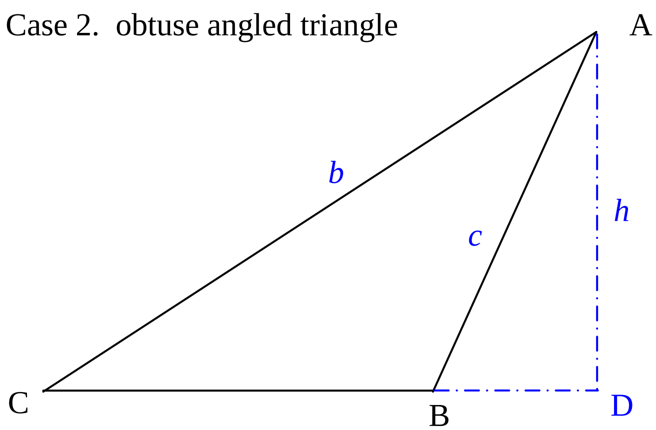
Set ①=②

$$b\sin C = c\sin B$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Rotate and repeat for the full Sine Law:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Case 2. obtuse angled triangle



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In ΔADB,

$$\sin B = \frac{h}{c}$$
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Set ①=②

$$b\sin C = c\sin B$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Rotate and repeat for the full Sine Law:

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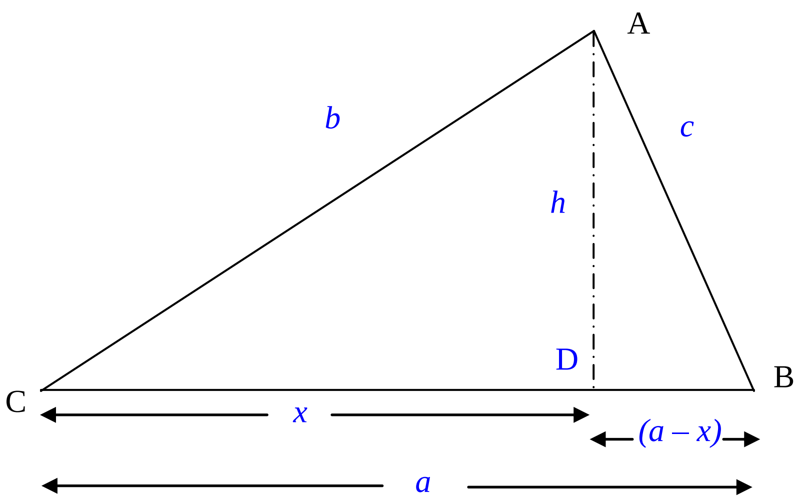
Derive the formula for Cosine Law.



In ΔACD,

$x^2 + h^2 = b^2$

or $h^2 = b^2 - x^2$ ①

and

$\cos C = \frac{x}{b}$

$\therefore x = b\cos C$ ②

In ΔABD,

$h^2 + (a - x)^2 = c^2$ ③

Sub ① into ③:

$(b^2 - x^2) + (a - x)^2 = c^2$

$b^2 - x^2 + a^2 - 2ax + x^2 = c^2$

$a^2 + b^2 - 2ax = c^2$

Substituting ② we get,

$a^2 + b^2 - 2ab\cos C = c^2$