# Proving Rapid Sums Identities II (algebraic)

**Date:**

Prove:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Young Gauss already proved this for us by doing a reversal and doubling the series. It's a neat trick which leads to the more general formula for the sum of an arithmetic series.

Consider the expansion of $\sum_{i=1}^{n}(i+1)^3$ to prove $\sum_{i=1}^{n} i^2$.

$$\sum_{i=1}^{n}(i+1)^3 = \sum_{i=1}^{n}(i+1)(i+1)(i+1)$$

Expand $(i+1)^3$

Isolate $\sum i^2$. (and it will involve telescoping series!)

⋮

$$\therefore \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Consider the expansion of $\sum_{i=1}^{n}(i+1)^4$ to prove $\sum_{i=1}^{n} i^3$.

$$\sum_{i=1}^{n}(i+1)^4 = \sum_{i=1}^{n}\left(i^4+4i^3+6i^2+4i+1\right)$$

$$= \sum_{i=1}^{n} i^4 + 4\sum_{i=1}^{n} i^3 + 6\sum_{i=1}^{n} i^2 + 4\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

Isolate, because this is what we're trying to prove.