Properties of Sigma Notation

Date:

Instructions:

What follows below is a list of possible properties of sigma notation. Some of them are correct for all $n \geq 1$, $n \in \mathbb{N}$, and thus are **properties** or **identities** of sigma notation. Others are not <u>always</u> true, and thus it is incorrect to call them identities.

1. For each of the following statements, let ***n = 3 (except for #7 and #8 where m = 2 and n = 5)*** and write in your notebooks the sums represented by the left and right hand sigma's explicitly. If the left and right hand sides are <u>not</u> equal, then label the property ***"FALSE"*** on the table below.

2. For those statements <u>not</u> labelled "false" in question #1, attempt to prove each one by writing both the left and right hand sides explicitly in your notebooks and then showing them to be equal.
For example:

$$\sum_{i=1}^{n} x^i \quad \text{would be written explicitly as :} \quad x + x^2 + x^3 + \ldots + x^{n-1} + x^n$$

1. Independence from Choice of Dummy Variable: $\sum_{i=1}^{n} x^i = \sum_{k=1}^{n} x^k$	***2. Constant Factor Common to all Terms:*** $\sum_{i=1}^{n} cx^i = c\sum_{i=1}^{n} x^i$
3. All Terms Constant: $\sum_{i=1}^{n} 6 = 6n$ or more generally : $\sum_{i=1}^{n} c = cn$	***4. Multiple Term Arguments:*** $\sum_{i=1}^{n} \left(x^i + y^i\right) = \sum_{i=1}^{n} x^i + \sum_{i=1}^{n} y^i$
5. Sigma of Products: $\sum_{i=1}^{n} x^i y^i = \left(\sum_{i=1}^{n} x^i\right)\left(\sum_{i=1}^{n} y^i\right)$	***6. Reciprocal Property:*** $\sum_{i=1}^{n} \frac{1}{x^i} = \frac{1}{\sum_{i=1}^{n} x^i}$
7. Changing Limits of Summation: $\sum_{i=m}^{n} x^i = \sum_{i=1}^{n-(m-1)} x^{i+m-1}$ where $1 < m < n$	***8. Sequential Sums:*** $\sum_{i=1}^{n} x^i = \sum_{i=1}^{m} x^i + \sum_{i=m+1}^{n} x^i$ where $1 < m < n$
9. Reversing Order of Terms: $\sum_{i=0}^{n} x^{n-i} y^i = \sum_{i=0}^{n} x^i y^{n-i}$	***10. Telescoping Sums:*** $\sum_{i=1}^{n} \left(x^{i+1} - x^i\right) = x^{n+1} - x$

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Use the property of Sigma notation named in each question to rewrite each given expression in an alternate form.

1. Constant Factor Common to all Terms:

a) $\sum_{i=1}^{6} 3i^2 =$ b) $\sum_{i=1}^{6} hijk =$ c) $\sum_{i=5}^{n} abi \sin i \cos k =$

2. All Terms Constant:

a) $\sum_{i=1}^{d} abc =$ b) $\sum_{i=1}^{m} \frac{x^y}{z} =$ c) $\sum_{i=a}^{b} c =$

3. ***Sequential Sums:*** *(Complete the missing arguments and limits of summation.)*

a) $\sum_{i=1}^{100} 3i^4 = \sum_{i=1}^{20} 3i^4 + \sum$

b) $\sum_{i=50}^{120} 2ix^i = \sum_{i=1} 2ix^i - \sum$

c) $\sum_{i=1}^{56} (3i-7) = \sum_{i=1}^{88} (3i-7) - \sum$

d) $\sum_{i=7}^{39} i^i = \sum_{i=7} i^i + \sum_{i=23} i^i$

e) $\sum_{i=66}^{99} \sin i = \sum_{i=54}^{120} \sin i - \sum \sin i - \sum \sin i$

f) $\sum_{i=-14}^{44} \frac{i}{3} = \sum_{i=0}^{60} \frac{i}{3} + \sum \frac{i}{3} - \sum \frac{i}{3}$

4. ***Reversing Order of Terms:*** *(Hint: It may help to write the series explicitly as a middle step.)*
Note: In each case keep the limits of summations the same as in the question.

a) $\sum_{i=0}^{25} i =$

b) $\sum_{i=1}^{25} i =$

c) $\sum_{i=5}^{25} i =$

d) $\sum_{i=-2}^{25} i =$

e) $\sum_{i=0}^{20} (3i+2) =$

f) $\sum_{i=1}^{20} (3i+2) =$

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Now try to write a more general formula for reversing the order of terms when the lower limit of summation is not equal to zero.

$$\sum_{i=m}^{n} x^i =$$

Now try to reverse the terms using the above formula and check your answer by writing the series explicity (both questions and answers).

g) $\sum_{i=10}^{20}(3i+2)=$

h) $\sum_{i=-2}^{100} i^2 =$

i) $\sum_{i=1}^{5}(40-5i)=$

5. ***Telescoping Sums:*** *(Write the first four expressions explicitly as a middle step. — Try the last four problems* ***without*** *first writing them explicitly.)* ***Simplify where possible.***

a) $\sum_{i=1}^{4}\left(x^i - x^{i-1}\right)=$

b) $\sum_{i=0}^{2}\left((i+1)-i\right)=$

c) $\sum_{i=1}^{3}\left((i+2)^2-(i+1)^2\right)=$

d) $\sum_{i=4}^{7}\left(i^5-(i-1)^5\right)=$

e) $\sum_{i=4}^{49}\left(5^{i+1}-5^i\right)=$

f) $\sum_{i=-2}^{11}\left(\frac{3}{i+4}-\frac{3}{i+3}\right)=$

g) $\sum_{i=10}^{k}\left((i+1)^i-(i)^{i-1}\right)=$

h) $\sum_{i=k-3}^{2k+4}\left(\sin(i+10)-\sin(i+9)\right)=$