Properties of Sigma Notation (proofs)

Date:

1. Independence from Choice of Dummy Variable:

$$\sum_{i=1}^{n} x^i = \sum_{k=1}^{n} x^k$$

$LS : \sum_{i=1}^{n} x^i$

$= 1 + 2 + 3 + \ldots + n$

$RS : \sum_{k=1}^{n} x^k$

$= 1 + 2 + 3 + \ldots + n$

Therefore, LS = RS.

2. Constant Factor Common to all Terms:

$$\sum_{i=1}^{n} cx^i = c\sum_{i=1}^{n} x^i$$

$LS : \sum_{i=1}^{n} cx^i$

$= cx^1 + cx^2 + cx^3 + \ldots + cx^n$

$= c\left(x^1 + x^2 + x^3 + \ldots + x^n\right)$

$RS : c\sum_{i=1}^{n} x^i$

$= c\left(x^1 + x^2 + x^3 + \ldots + x^n\right)$

Therefore, LS = RS.

3. All Terms Constant:

$$\sum_{i=1}^{n} 6 = 6n \quad \text{or more generally} \quad : \quad \sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} c = \underbrace{c + c + c + \ldots + c}_{n \text{ times}} = cn$$

4. Multiple Term Arguments:

$$\sum_{i=1}^{n} \left(x^i + y^i\right) = \sum_{i=1}^{n} x^i + \sum_{i=1}^{n} y^i$$

$\sum_{i=1}^{n} \left(x^i + y^i\right)$

$= (x + y) + \left(x^2 + y^2\right) + \left(x^3 + y^3\right) + \ldots + \left(x^n + y^n\right)$

$= \left(x + x^2 + x^3 + \ldots + x^n\right) + \left(y + y^2 + y^3 + \ldots + y^n\right)$

$= \sum_{i=1}^{n} x^i + \sum_{i=1}^{n} y^i$

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5. Sigma of Products:

$$\sum_{i=1}^{n} x^i y^i = \left(\sum_{i=1}^{n} x^i\right)\left(\sum_{i=1}^{n} y^i\right)$$ **FALSE**

Let $n = 2$.

$LS: \sum_{i=1}^{2} x^i y^i$

$= xy + x^2y^2$

$RS: \left(\sum_{i=1}^{2} x^i\right)\left(\sum_{i=1}^{2} y^i\right)$

$= (x + x^2)(y + y^2)$

$= xy + xy^2 + x^2y + x^2y^2$

Therefore, LS ≠ RS.

6. Reciprocal Property:

$$\sum_{i=1}^{n} \frac{1}{x^i} = \frac{1}{\sum_{i=1}^{n} x^i}$$ **FALSE**

$LS: \sum_{i=1}^{2} \frac{1}{x^i}$

$= \frac{1}{x^1} + \frac{1}{x^2}$

$RS: \frac{1}{\sum_{i=1}^{2} x^i}$

$= \frac{1}{x + x^2}$

Therefore, LS ≠ RS.

7. Changing Limits of Summation:

$$\sum_{i=m}^{n} x^i = \sum_{i=1}^{n-(m-1)} x^{i+m-1} \quad \text{where} \quad 1 < m < n$$

$LS: \sum_{i=m}^{n} x^i$ where $1 < m < n$

$= x^m + x^{m+1} + x^{m+2} + \ldots + x^n$

$RS: \sum_{i=1}^{n-(m-1)} x^{i+m-1}$ where $1 < m < n$

$= x^{1+m-1} + x^{2+m-1} + x^{3+m-1} + \ldots + x^{n-(m-1)+m-1}$

$= x^m + x^{m+1} + x^{m+2} + \ldots + x^n$

Therefore, LS = RS.

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8. Sequential Sums:

$$\sum_{i=1}^{n} x^i = \sum_{i=1}^{m} x^i + \sum_{i=m+1}^{n} x^i \quad \text{where} \quad 1 < m < n$$

$$\sum_{i=1}^{n} x^i$$

$$= \underbrace{x + x^2 + x^3 + \ldots + x^{m-1} + x^m}_{m \text{ terms}} + \underbrace{x^{m+1} + \cdots + x^n}, \text{ where } 1 < m < n$$

$$= \sum_{i=1}^{m} x^i + \sum_{i=m+1}^{n} x^i \quad \text{where } 1 < m < n$$

9. Reversing Order of Terms:

$$\sum_{i=0}^{n} x^{n-i} y^i = \sum_{i=0}^{n} x^i y^{n-i}$$

$$\sum_{i=0}^{n} x^{n-i} y^i$$

$$= x^n y^0 + x^{n-1} y^1 + x^{n-2} y^2 + x^{n-3} y^3 + \ldots + x^1 y^{n-1} + x^0 y^n$$

$$= x^0 y^n + x^1 y^{n-1} + \ldots + x^{n-2} y^2 + x^{n-1} y^1 + x^n y^0 \quad \text{(reverse terms)}$$

$$= \sum_{i=0}^{n} x^i y^{n-i}$$

10. Telescoping Sums:

$$\sum_{i=1}^{n} \left(x^{i+1} - x^i\right) = x^{n+1} - x$$

$$\sum_{i=1}^{n} \left(x^{i+1} - x^i\right)$$

$$= \left(x^2 - x^1\right) + \left(x^3 - x^2\right) + \left(x^4 - x^3\right) + \ldots + \left(x^n - x^{n-1}\right) + \left(x^{n+1} - x^n\right)$$

$$= -x^1 + \underbrace{x^2 - x^2 + x^3 - x^3 + -}_{\text{all cancel}} \ldots + x^{n+1}$$

$$= x^{n+1} - x$$