Properties of Sigma Notation (Answers)

Date:

Instructions:

What follows below is a list of possible properties of sigma notation. Some of them are correct for all $n \geq 1, n \in \mathbb{N}$, and thus are **properties** or **identities** of sigma notation. Others are not always true, and thus it is incorrect to call them identities.

1. For each of the following statements, let ***n = 3 (except for #7 and #8 where m = 2 and n = 5)*** and write in your notebooks the sums represented by the left and right hand sigma's explicitly. If the left and right hand sides are not equal, then label the property ***"FALSE"*** on the table below.
2. For those statements not labelled "false" in question #1, attempt to prove each one by writing both the left and right hand sides explicitly in your notebooks and then showing them to be equal.
   For example:

$$\sum_{i=1}^{n} x^i \text{ would be written explicitly as : } x + x^2 + x^3 + \ldots + x^{n-1} + x^n$$

1. Independence from Choice of Dummy Variable: $\sum_{i=1}^{n} x^i = \sum_{k=1}^{n} x^k$	***2. Constant Factor Common to all Terms:*** $\sum_{i=1}^{n} cx^i = c\sum_{i=1}^{n} x^i$
3. All Terms Constant: $\sum_{i=1}^{n} 6 = 6n$ or more generally : $\sum_{i=1}^{n} c = cn$	***4. Multiple Term Arguments:*** $\sum_{i=1}^{n} \left(x^i + y^i\right) = \sum_{i=1}^{n} x^i + \sum_{i=1}^{n} y^i$
5. Sigma of Products: $\sum_{i=1}^{n} x^i y^i = \left(\sum_{i=1}^{n} x^i\right)\left(\sum_{i=1}^{n} y^i\right)$ **FALSE**	***6. Reciprocal Property:*** $\sum_{i=1}^{n} \frac{1}{x^i} = \frac{1}{\sum_{i=1}^{n} x^i}$ **FALSE**
7. Changing Limits of Summation: $\sum_{i=m}^{n} x^i = \sum_{i=1}^{n-(m-1)} x^{i+m-1}$ where $1 < m < n$	***8. Sequential Sums:*** $\sum_{i=1}^{n} x^i = \sum_{i=1}^{m} x^i + \sum_{i=m+1}^{n} x^i$ where $1 < m < n$
9. Reversing Order of Terms: $\sum_{i=0}^{n} x^{n-i} y^i = \sum_{i=0}^{n} x^i y^{n-i}$	***10. Telescoping Sums:*** $\sum_{i=1}^{n} \left(x^{i+1} - x^i\right) = x^{n+1} - x$

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Use the property of Sigma notation named in each question to rewrite each given expression in an alternate form.

1. Constant Factor Common to all Terms:

a) $\sum_{i=1}^{6} 3i^2 = 3\sum_{i=1}^{6} i^2$ b) $\sum_{i=1}^{6} hijk = hjk\sum_{i=1}^{6} i$ c) $\sum_{i=5}^{n} abi\sin i\cos k = ab\cos k\sum_{i=5}^{n} i\sin i$

2. All Terms Constant:

a) $\sum_{i=1}^{d} abc = abcd$ b) $\sum_{i=1}^{m} \frac{x^y}{z} = \frac{mx^y}{z}$ c) $\sum_{i=a}^{b} c = c(b-a+1)$

3. ***Sequential Sums:*** *(Complete the missing arguments and limits of summation.)*

a) $\sum_{i=1}^{100} 3i^4 = \sum_{i=1}^{20} 3i^4 + \sum_{i=21}^{100} 3i^4$ b) $\sum_{i=50}^{120} 2ix^i = \sum_{i=1}^{120} 2ix^i - \sum_{i=1}^{49} 2ix^i$

c) $\sum_{i=1}^{56} (3i-7) = \sum_{i=1}^{88} (3i-7) - \sum_{i=57}^{88} (3i-7)$ d) $\sum_{i=7}^{39} i^i = \sum_{i=7}^{22} i^i + \sum_{i=23}^{39} i^i$

e) $\sum_{i=66}^{99} \sin i = \sum_{i=54}^{120} \sin i - \sum_{i=54}^{65} \sin i - \sum_{i=100}^{120} \sin i$ f) $\sum_{i=-14}^{44} \frac{i}{3} = \sum_{i=0}^{60} \frac{i}{3} + \sum_{i=-14}^{-1} \frac{i}{3} - \sum_{i=45}^{60} \frac{i}{3}$

4. ***Reversing Order of Terms:*** *(Hint: It may help to write the series explicitly as a middle step.)*
Note: In each case keep the limits of summations the same as in the question.

a) $\sum_{i=0}^{25} i = 0+1+2+3+\ldots+24+25 = 25+24+\ldots+2+1+0 = \sum_{i=0}^{25} (25-i)$

b) $\sum_{i=1}^{25} i = 1+2+3\ldots+24+25 = 25+24+\ldots+3+2+1 = \sum_{i=1}^{25} (26-i)$

c) $\sum_{i=5}^{25} i = 5+6+7+\ldots+24+25 = 25+24+\ldots+7+6+5 = \sum_{i=5}^{25} (30-i)$

d) $\sum_{i=-2}^{25} i = -2-1+0+1+2+\ldots+24+25 = 25+24+\ldots+-1-2 = \sum_{i=-2}^{25} (23-i)$

e) $\sum_{i=0}^{20} (3i+2) = \cdots = \sum_{i=0}^{20} (62-3i)$

f) $\sum_{i=1}^{20} (3i+2) = \ldots = \sum_{i=1}^{20} (65-3i)$

Now try to write a more general formula for reversing the order of terms when the lower limit of summation is not equal to zero.

$$\sum_{i=m}^{n} x^i = \sum_{i=m}^{n} x^{(n+m-i)}$$

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Now try to reverse the terms using the above formula and check your answer by writing the series explicity (both questions and answers).

g) $\sum_{i=10}^{20}(3i+2)=\sum_{i=10}^{20}(92-3i)$

h) $\sum_{i=-2}^{100}i^2=\sum_{i=-2}^{100}(100+(-2)-i)^2=\sum_{i=-2}^{100}(98-i)^2$

i) $\sum_{i=1}^{5}(40-5i)=\sum_{i=1}^{5}(10+5i)$

5. ***Telescoping Sums:*** *(Write the first four expressions explicitly as a middle step. — Try the last four problems* ***without*** *first writing them explicitly.)* ***Simplify where possible.***

a) $\sum_{i=1}^{4}\left(x^i-x^{i-1}\right)=x-1+x^2-x+x^3-x^2+x^4-x^3=x^4-1$

b) $\sum_{i=0}^{2}((i+1)-i)=1-0+2-1+3-2=3$

c) $\sum_{i=1}^{3}\left((i+2)^2-(i+1)^2\right)=3^2-2^2+4^2-3^2+5^2-4^2=5^2-2^2=21$

d) $\sum_{i=4}^{7}\left(i^5-(i-1)^5\right)=4^5-3^5+5^5-4^5+6^5-5^5+7^5-6^5=7^5-3^5=16\,564$

e) $\sum_{i=4}^{49}\left(5^{i+1}-5^i\right)=5^5-5^4+5^6-5^5+5^7-5^6+\ldots+5^{50}-5^{49}=5^{50}-5^4$

f) $\sum_{i=-2}^{11}\left(\frac{3}{i+4}-\frac{3}{i+3}\right)=\frac{3}{11+4}-\frac{2}{-2+3}=\frac{-14}{5}$

g) $\sum_{i=10}^{k}\left((i+1)^i-(i)^{i-1}\right)=(k+1)^k-10^9$

h) $\sum_{i=k-3}^{2k+4}(\sin(i+10)-\sin(i+9))=\sin(2k+14)-\sin(k+6)$