Maclaurin Series

Date:

Add to your list of mathematical heroes: Brook Taylor (1685-1731) and Colin Maclaurin (1698-1746)

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{r=0}^{\infty} \frac{(-1)^r x^{(2r+1)}}{(2r+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{r=0}^{\infty} \frac{(-1)^r x^{(2r)}}{(2r)!}$$

Calculators use Maclaurin series to calculate numerically rather than searching databases of values. For additional information, research Taylor series and Maclaurin series.

Exercises

1. Graph $y = \sin x$, zoomed into $-\pi \le x \le \pi$.

   Compare the graph of $y = \sin x$ with each of:

   a) $f(x) = \frac{x}{1!}$

   b) $g(x) = \frac{x}{1!} - \frac{x^3}{3!}$

   c) $h(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!}$

   d) $i(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

   e) $j(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

2. Graph $y = \cos x$, $-\pi \le x \le \pi$.

   Compare the graph of $y = \cos x$ with each of:

   a) $f(x) = 1 - \frac{x^2}{2!}$

   b) $g(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

   c) $h(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

   d) $i(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$

   e) $j(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$

3. What do you notice about each approximation function with an additional term is added to the series? (Compare with the previous approximation function. Compare with the original trig function.)

   As the series approaches infinity, the approximation series will converge ***exactly*** to the trig function. Computers use these series to calculate values because it is faster to calculate in a loop than looking up values in a table. Explaining this concept is beyond our abilities right now, but we will get to it one day. ☺