Introduction to Sigma Notation (homework)

Date:

1. Express each of the series below in sigma notation using the most convenient lower limit of summation. Do not use a lower limit of 1.

a) $4 + 5 + 6 + 7 + 8 + \ldots + 34$

b) $-4 - 3 - 2 - 1 + 0 + \ldots$ (to 25 terms)

c) $\frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7} + \frac{7}{8}$

d) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

2. Express each of the series below in sigma notation with a lower limit of summation of 1.

a) $4 + 5 + 6 + 7 + 8 + \ldots + 34$

b) $-4 - 3 - 2 - 1 + 0 + \ldots$ (to 25 terms)

c) $6 + 12 + 18 + 24 + \ldots + 150$

d) $7 + 13 + 19 + 25 + \ldots + 151$

e) $4 + 10 + 16 + 22 + \ldots + 148$

f) $1 + 8 + 15 + 22 + \ldots$ (to n terms)

g) $a^b + a^{2b} + a^{3b} + \ldots + a^{nb}$

h) $9 + 4 - 1 - 6 - \ldots + (14 - 5n)$

i) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

j) $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{2n}$

k) $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$

l) $4 - 8 + 12 - 16 + \ldots - 40$

m) $-2 - 4 - 6 - 8 - \ldots - 142$

n) $\frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7} + \frac{7}{8}$

o) $12 + 32 + 52 + 72 + \ldots$ (to 2n terms)

p) $-9 + 13 - 17 + 21 - \ldots$ (to n–1 terms)

q) $(4)(10) + (8)(15) + (12)(20) + (16)(25) + \ldots + (120)(155)$

r) $\frac{1}{2\times 3} + \frac{1}{3\times 6} + \frac{1}{4\times 9} + \frac{1}{5\times 12} + \ldots + \frac{1}{10\times 27}$

3. Express each of the series below in sigma notation with a lower limit of summation of 2.

a) $4 + 5 + 6 + 7 + 8 + \ldots + 34$

b) $-4 - 3 - 2 - 1 + 0 + \ldots$ (to 25 terms)

c) $\frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7} + \frac{7}{8}$

d) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

e) $12 + 32 + 52 + 72 + \ldots$ (to $2n$ terms)

f) $4(10) + 8(15) + 12(20) + \ldots + 120(155)$

4. Notice that questions #3 to #5 have many questions in common. You have had the opportunity to express some of the series in two or three different sigma notations. By comparing different answers for the same series, develop a rule for calculating the correct new argument when the limits of summation have been changed. Use this rule to answer #7.

5. Change the lower limit of summation for each of the following to "1".

a) $\sum_{i=5}^{20}(i+3)$

b) $\sum_{i=50}^{75}(2i+3)$

c) $\sum_{i=-5}^{27}(5-3i)$

d) $\sum_{i=0}^{7}i^2$

e) $\sum_{i=-1}^{100}(i-4)(i+3)$

f) $\sum_{i=-35}^{-20}8i$

g) $\sum_{i=12}^{30}\left(i^2-3i+7\right)$

Answers:

1. a) $\sum_{i=4}^{34} i$ b) $\sum_{i=-4}^{20} i$ c) $\sum_{i=4}^{8}(-1)^i\left(\frac{i-1}{i}\right)$ or $\sum_{i=3}^{7}(-1)^{i+1}\left(\frac{i}{i+1}\right)$ d) $\sum_{i=0}^{4}\frac{1}{3^i}$

2. a) $\sum_{i=1}^{31}(i+3)$ b) $\sum_{i=1}^{25}(i-5)$ c) $\sum_{i=1}^{25}6i$ d) $\sum_{i=1}^{25}(6i+1)$ e) $\sum_{i=1}^{25}(6i-2)$

   f) $\sum_{i=1}^{n}(7i-6)$ g) $\sum_{i=1}^{n}a^{ib}$ h) $\sum_{i=1}^{n}(14-5i)$ i) $\sum_{i=1}^{5}\frac{1}{3^{i-1}}$ j) $\sum_{i=1}^{2n}\frac{1}{i}$

   k) $\sum_{i=1}^{n}\frac{1}{n+i}$ l) $\sum_{i=1}^{10}(-1)^{i-1}4i$ m) $\sum_{i=1}^{71}(-2i)$ n) $\sum_{i=1}^{5}(-1)^{i+1}\left(\frac{i+2}{i+3}\right)$ o) $\sum_{i=1}^{2n}(2i-1)^2$

   p) $\sum_{i=1}^{n-1}(-1)^i(4i+5)$ q) $\sum_{i=1}^{30}(4i)(5i+5)$ r) $\sum_{i=1}^{9}\frac{1}{(i+1)(3i)}$ or $\sum_{i=1}^{9}\frac{1}{3i(i+1)}$

3. a) $\sum_{i=2}^{32}(i+2)$ b) $\sum_{i=2}^{26}(i-6)$ c) $\sum_{i=2}^{6}(-1)^i\left(\frac{i+1}{i+2}\right)$ d) $\sum_{i=2}^{6}\frac{1}{3^{i-2}}$ e) $\sum_{i=2}^{2n+1}(2i-3)^2$ f) $\sum_{i=2}^{31}(4i-4)(5i)$

5. a) $\sum_{i=5}^{20}(i+3)=\sum_{i=5-4}^{20-4}\left(\left(i+4\right)+3\right)=\sum_{i=1}^{16}(i+7)$

   b) $\sum_{i=50}^{75}(2i+3)=\sum_{i=50-49}^{75-49}\left(2\left(i+49\right)+3\right)=\sum_{i=1}^{26}(2i+101)$

   c) $\sum_{i=-5}^{27}(5-3i)=\sum_{i=-5+6}^{27+6}\left(5-3\left(i-6\right)\right)=\sum_{i=1}^{33}(23-3i)$

   d) $\sum_{i=0}^{7}i^2=\sum_{i=1}^{8}(i-1)^2$ *or* $\sum_{i=1}^{8}\left(i^2-2i+1\right)$

   e) $\sum_{i=-1}^{100}(i-4)(i+3)=\sum_{i=-1+2}^{100+2}\left(\left(i-2\right)-4\right)\left(\left(i-2\right)+3\right)=\sum_{i=1}^{102}(i-6)(i+1)$ *or* $\sum_{i=1}^{102}\left(i^2-5i-6\right)$

   f) $\sum_{i=-35}^{-20}8i=\sum_{i=-35+36}^{-20+36}8(i-36)=\sum_{i=1}^{16}(8i-288)$

   g) $\sum_{i=12}^{30}\left(i^2-3i+7\right)=\sum_{i=12-11}^{30-11}\left(\left(i+11\right)^2-3\left(i+11\right)+7\right)$

   $=\sum_{i=1}^{19}\left(i^2+22i+121-3i-33+7\right)$

   $=\sum_{i=1}^{19}\left(i^2+19i+95\right)$