Infinite Geometric Series (homework)

Date:

A **1.** Using the formula for the sum of an infinite geometric series, we deduce that $1 + 2 + 4 + 8 + 16 + \ldots = -1$. What is wrong?

2. We can "prove" that $1 - 1 + 1 - 1 + \ldots = \frac{1}{2}$. How? What is the error in this argument?

B **3.** Evaluate the following:

(a) $\sum_{i=1}^{\infty} 2(3)^{-i}$ (b) $\sum_{i=1}^{\infty} \left(-\frac{1}{2}\right)^{i-1}$

(c) $\sum_{i=1}^{\infty} 8(2)^{-i-1}$ (d) $5 + 4.5 + 4.05 + \ldots$

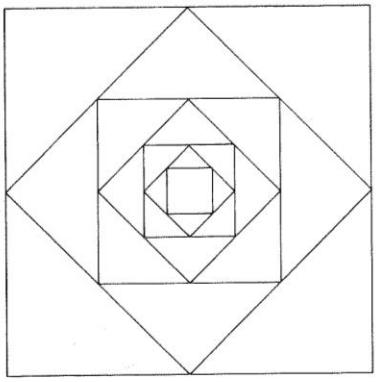
(e) $\frac{1}{3} + \frac{1}{4} + \frac{3}{16} + \ldots$ (f) $5 + 2.5 + 1.25 + 0.625 + \ldots$

4. Express the following repeating decimals as common fractions

(a) $0.\overline{63}$ (b) $0.6\overline{72}$

(c) $0.1\overline{853}$ (d) $0.\overline{485\ 714\ 2}$

5. Show that $1.\overline{0} = 0.\overline{9}$.



6. A square with 1 cm sides is drawn. A second square is drawn having as its vertices the midpoints of the sides of the front square; a third square is drawn having as its vertices the midpoints of the second square; and so on.

(a) Find the area of the 10th square.

(b) Show that the sum of the lengths of the sides of all the squares is $2 + \sqrt{2}$ cm.

7. Suppose $s > 0$. Evaluate $(1+s)^{-1} + (1+s)^{-2} + (1+s)^{-3} + \ldots$.

8. When, between five and six o'clock, are the hands of a clock together?

Answers

1. The formula only applies if the common ratio, r, satisfies $|r| < 1$.

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3. (a) 1 (b) $\frac{2}{3}$ (c) 4 (d) 50 (e) $\frac{4}{3}$ (f) 10

4. (a) $\frac{7}{11}$ (b) $\frac{37}{55}$ (c) $\frac{926}{4995}$ (d) $\frac{17}{35}$

6. (a) $\frac{1}{2^9}$ **7.** s^{-1}

8. $27\frac{3}{11}$ minutes after five o'clock