Equivalent Rates

Date:

A problem that is encountered with annuities concerns different compounding and payment periods. The annuity formulas can only be applied when these periods are the same.

Amount of an annuity formula, $\mathbf{A = \frac{R((1+i)^n - 1)}{i}}$

Present value of an annuity formula, $\mathbf{PV = \frac{R(1-(1+i)^{-n})}{i}}$

However we can solve this problem by using "**equivalent rates**" to change the given interest compounding period into an interest compounding period that will match the payment period.

Two interest rates are said to be **equivalent** if they yield the same amount of interest over the same time.

Example 1:

Suppose $1 is deposited in an account that earns 10%/a, compounded annually. Also suppose that $1 is deposited in an account that earns interest compounded quarterly. What interest rate would be required for the second deposit so that the value of each account is the same?

Account 1—annually	*Account 2—quarterly*
P = 1	*P = 1*
i = 10%	*i = ?*
n = 1	*n = 4*
$\therefore A = 1(1.1)^1$	$\therefore A = 1(1+i)^4$

Since these two amounts must be the same, equate them and solve for 'i'.

$$1(1.1)^1 = 1(1+i)^4$$
$$\sqrt[4]{1.1} = 1+i$$
$$\sqrt[4]{1.1} - 1 = i$$
$$i \doteq 0.0241137$$

You need several decimal places of accuracy here.
** Remember this is the rate per quarter.

$\therefore$ An **annual** rate of **4 x i** or 9.65% compounded quarterly is equivalent to 10%/a compounded annually.

Remember – invest $1, over 1 year, and solve.

Example 2:

Convert 3.8%/a compounded semi annually to an equivalent monthly rate.

Example 3:

A man has been making a series of 20 semi-annual deposits of $750 into an account that pays him interest of 3.8%/a compounded quarterly. What will the value of his account be when the last deposit is made?